Effects of Monetary and Fiscal Policy on the Yield Curve: Evidence from the U.S. and U.K.*

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Abstract

Monetary authorities in the U.S. and U.K. commenced special program purchases of longer-term government bonds in March 2009. One of the purposes of the programs was to influence the prices of assets acquired, that is, to lower middle- and long-term interest rates. The U.K. program played a role in that direction, but the U.S. program appears to have failed to attain its objective.

Can the central bank depress middle- and long-term interest rates by acquiring middle- and long-term government bonds, while issues of the corresponding bond increase? Using a New-Keynesian model, this study examines the relationship between monetary policy and the yield curve, considering the influence of government bond issues on the yield as well. As per the simulation results, whether the central bank can influence the middle point and/or the long end of the yield curve by purchasing the corresponding bonds would depend on the balance between the bonds acquired by the central bank and those issued by the government or the increase in the outstandings of bonds with corresponding terms to maturity.

The ratios of government bond purchases by the monetary authority to both bond issues and average outstandings are much higher in the U.K. than in the U.S. This confirms the empirical results of this study.

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Contents

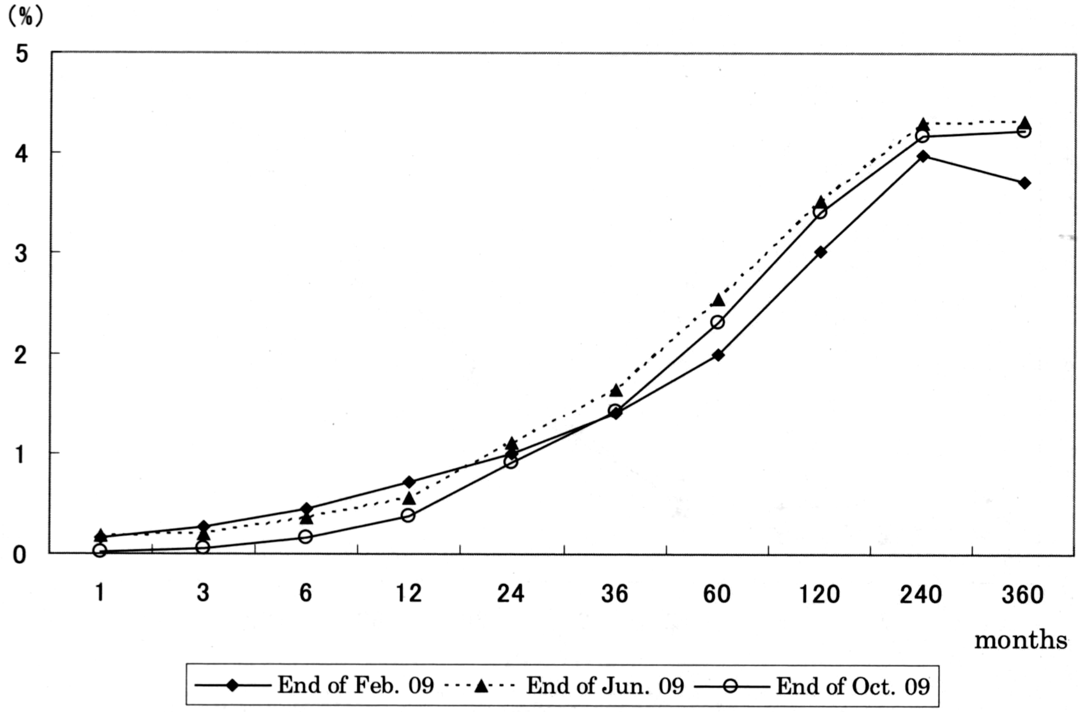
Ⅰ Introduction

Since the Lehman Brothers Holding Inc. filed for bankruptcy on September 15, 2008, monetary authorities in major countries have put in place a wide range of measures to provide liquidity to the economy, mainly in the financial markets – one such measure being the purchase of government securities from different financial institutions. Monetary authorities in both the U.S. and U.K., too, adopted this course.

In the U.S., the Federal Reserve Board (FRB) started to purchase middle- and long-term Treasury securities in March 2009 under a special program (longer-dated Treasury purchase program) and completed the program at the end of October 2009. Eligible securities under the program are Treasury notes, Treasury bonds and Treasury Inflation-Protected Securities (TIPS); the program mainly targeted securities with terms to maturity of 2 to 10 years[1)]. The FRB acquired Treasury securities of 300 billion dollars during the period. Meanwhile, in the U.K., the Bank of England Asset Purchase Facility Fund Limited (BEAPFF) also commenced, in March 2009, purchases of conventional gilts with residual maturities of longer than 3 years, under the Asset Purchase Facility (APF), and suspended purchases at the end of January 2010[2)]. The BEAPFF acquired gilts of around 198 billion pounds during the period.

Although the main purpose of these programs is to improve liquidity in the credit market, they also aim to influence the prices of the assets purchased, that is, to lower middle- and long- term interest rates in order to encourage investments in the private sector[3)]. Have these special program purchases of government securities depressed middle- and long-term interest rates? Figures 1 and 2 demonstrate changes in the yield curve from the starting point of the programs to

Figure 1. U.S. Yield Curve



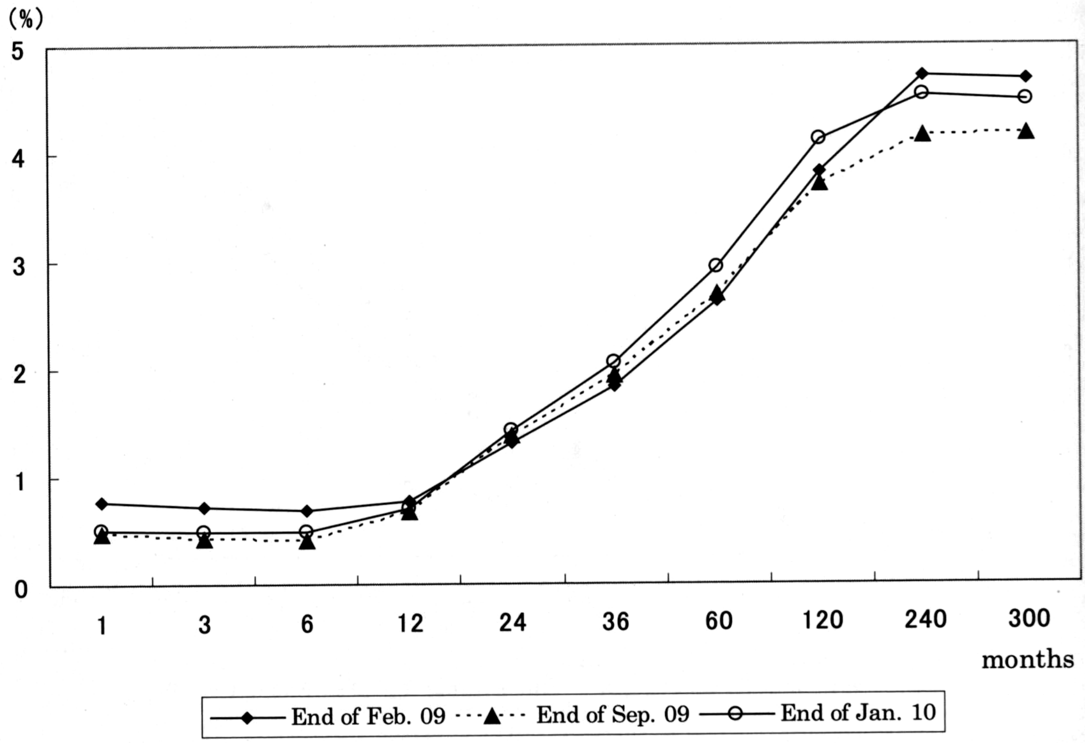
Source: *U.S. Department of the Treasury's web site.*

the end or suspending point for the U.S. and U.K., respectively. Figure 1 indicates that the program appears to have failed to make the intended impact on the U.S. yield curve, especially on yields with residual maturities of more than 5 years. With respect to the long end above 10 years, higher yields might be because this range is the minor target of the program. On the other hand, Figure 2 shows that the program has played a role in lowering the U.K. yield curve, mainly in the long end. At the end of January 2010, the middle part of the yield curve rises, but the long end remains lower than at the end of February 2009.

Could monetary policy influence the yield curve, especially in the middle part and the long end? Many studies have examined the relationship between monetary policy and the term structure of interest rates. To our knowledge, the models in these studies as a whole might be categorized into three groups[4)]. The first group uses a single equation (partial equilibrium) model, which includes variables intended to capture the maturity distribution of central bank holdings and outstandings of government bonds, in order to explain bond yields or its term premia. Recent studies in the group are Kuttner [2006][5)] and Gagnon et al. [2010].

The second group resorts to a Vector Auto-Regression (VAR) model. Studies classified in this

Figure 2. U.K. Yield Curve



Source: *The Bank of England's web site*

group are Evans and Marshall [1998, 2007]; Bernanke, Reinhart and Sack [2004]; Diebold, Rudebusch and Aruoba [2006]; Lildholdt, Panigirtzoglou and Peacock [2007]; and Bianchi, Mumtaz and Surico [2009].

The last group uses a New-Keynesian dynamic stochastic general equilibrium (DSGE) model. Bekaert, Cho and Moreno [2005]; Ravenna and Seppälä [2005, 2006, 2007]; Rudebusch and Wu [2008]; Andrés, López-Salido and Nelson [2004a, b]; and Marzo, Söderström and Zagaglia [2007] belong to this group.

Most of these empirical studies find that responses of the yield curve to monetary policy shocks decline along the maturity structure. This implies that monetary policy could make some impacts on the middle point and on the long end of the yield curve. However these studies do not consider influences of changes in outstandings of government bonds (i.e., the supply of the government bonds) on the yield curve. That is, even if the central bank implements monetary easing policy by purchasing government bonds to lower middle- and long-term interest rates, larger-scale issues of middle- and long-term bonds would negate the lowering effect of the monetary policy.

Attaching importance to microfoundation of the model, this study uses a DSGE model based on the alternative approaches of Andrés, López-Salido and Nelson [2004a, b] (henceforth, ALSN) and Marzo, Söderström and Zagaglia [2007] (henceforth, MSZ), especially the former. As the literature in the third group, except for ALSN and MSZ, prices the term structure of interest rates using the kernel for one-period bonds extracted from the solution of the dynamic model, movements in term premia do not affect private agents' spending decisions. Consequently, the term premia depend on the economy, but there is no feedback in the opposite direction.

Although the basic structure of our model is based on the ALSN [2004a] approach, we introduce a radical revision to the framework of the monetary policy rule, so the acquisition of government bonds by the central bank has some influence on policy rate setting. Specifically, we link the central bank holdings of government bonds with the supply of high-powered money, which is incorporated into a rule for setting policy rates.

The purpose of this study is to examine the relationship between monetary policy and the yield curve, considering, at the same time, the influence of government bond issues on the yield curve. This paper is organized as follows. Section Ⅱ lays out the model. Section Ⅲ describes the determination of parameters in the model. We adopt a mix of calibration and estimation for deciding parameters. Thus, some parameters are set according to previous empirical studies, whereas others are estimated by the Bayesian inference method. This section explains the quantitative properties of the estimated parameter. Section Ⅳ provides simulation results indicating the responses of the yield curve to monetary policy and fiscal policy shocks. Section V concludes the paper.

Ⅱ Model setup

In this section, we lay out a model economy and display the problems solved by households and firms. In addition, we describe the behaviour of monetary and the fiscal authorities.

The starting point for our analysis is a hybrid-type New-Keynesian model with sticky prices and habits in consumption, but without capital accumulation. In addition, we evolve the standard hybrid New-Keynesian model by making two modifications as follows. First, it is assumed that there are three different types of government bonds - one-period or short-term bonds, middle-term bonds, and long-term bonds - and introduce a financial friction that makes these different types of bonds imperfect substitutes, as in ALSN [2004a, b] and MSZ [2007]. The friction brings about the endogenous term structure of interest rates in the sense that there exists bi-directional feedback between the yield curve and the economy.

Second, we assume that the supply of high-powered money is represented by a function of nominal outstandings of both middle- and long- term bonds, and incorporate the supply function of high-powered money into the central bank's policy function, in which the policy instrument is the one-period nominal interest rate. In the model it is presupposed that the central bank supplies high-powered money by purchasing middle- and/or long-term bonds; the effects of these purchases would be transferred to the endogenous term structure through the one-period rate set.

In the following paragraphs, we present the objective and constraints of different agents in the economy, paying special attention to the specification of household's preferences as well as of the central bank's policy function.

1. Households

(1) Utility function and budget constraint

We assume a continuum of identical and infinitely-lived households indexed by $i \in [0, 1]$, and a continuum of goods indexed by $j \in [0, 1]$ which the firm j produces. These households obtain utility from a bundle C_t given by

$$C_t=\left[\int_0^1 C_t(j)^{1-\frac{1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $C_t(j)$ represents the quantity of good j consumed by the household in period t, and ε the elasticity of substitution across different varieties of goods.

These households have a period utility function

$$U\left(C_t,\ C_{t-1},\ N_t,\ \frac{M_t}{P_t},\ \frac{M_{t-1}}{P_{t-1}},\ e_t\right)$$
$$=\frac{1}{1-\sigma}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma}+\frac{1}{1-\delta}\left(\frac{M_t}{e_tP_t}\right)^{1-\delta}-\frac{N_t^{1+\varphi}}{1+\varphi}-G\left(\frac{M_t}{P_t},\ \frac{M_{t-1}}{P_{t-1}}\right), \tag{2}$$

where N_t denotes hours worked by the household in period t; M_t/P_t, real balances of the household in period t; e_t, shocks to the household's demand for real balances in period t;$\sigma(>0)$, inverse of the elasticity of inter-temporal substitution; $\varphi(\geq 0)$, inverse of the Frisch labour supply elasticity; δ (>0), elasticity of the money demand; and $h(\geq 0)$, the habit persistence parameter indicating the extent of habit formation. The last term of the utility function represents portfolio adjustment cost, introduced in Nelson [2002] and ALSN [2004a, b], and is given by

$$G\left(\frac{M_t}{P_t},\ \frac{M_{t-1}}{P_{t-1}}\right)=\frac{d}{2}\left[\exp\left\{c\left(\frac{M_t/P_t}{M_{t-1}/P_{t-1}}-1\right)\right\}+\exp\left\{-c\left(\frac{M_t/P_t}{M_{t-1}/P_{t-1}}-1\right)\right\}-2\right], \tag{3}$$

where $d(>0)$ and $c(>0)$ are parameters. As shown below, due to the portfolio adjustment cost, the demand for real balances in period t depends not only on the real balances in period $t-1$ but also on the expected in period $t+1$.

The period budget constraint takes the form

$$\frac{1}{P_t}(R_{t-1}B_{s,t-1}+H_{m,t-1}B_{m,t-1}+H_{l,t-1}B_{l,t-1}+M_{t-1}+W_tN_t+TR_t+D_t)$$
$$=C_t+\frac{1}{P_t}\{B_{s,t}+B_{m,t}+B_{l,t}+M_t(1+AC_t)+TA_t\} \tag{4}$$

for $t = 0, 1, 2, \cdots$. P_t denotes the price of the consumption good; W_t, the nominal wage; TR_t, the lump-sum nominal transfer; D_t, the nominal dividend from firms; and TA_t, the nominal lump-sum tax in period t. R_t is the gross nominal return on the one-period bond in period t. $H_{k,t}$ stands for the gross one-period nominal holding returns on the middle-term bond for $k = m$ and the long-term bond for $k = l$ in period t. For a zero-coupon bond, the holding return is calculated by $H_{k,t}=\frac{E_tQ_{k,t+1}-Q_{k,t}}{Q_{k,t}}$, where $Q_{k,t}$ ($k = m$ and l) represents the price of the bond in period t. $B_{s,t}$ indicates one-period bond outstndings in period t and $B_{k,t}$ denotes middle- and long-term bonds outstandings in period t for $k = m$ and l, respectively. AC_t is the cost function for investing in middle- and long-term bonds, and is specified as follows:

$$AC_t\equiv\frac{v_m}{2}\left(\frac{M_t^{\theta}}{B_{m,t}}\kappa_m-1\right)^2+\frac{v_l}{2}\left(\frac{M_t^{1-\theta}}{B_{l,t}}\kappa_l-1\right)^2, \tag{5}$$

where v_m (>0), v_l (>0) and $\theta\in[0, 1]$ are parameters. κ_m (>0) and κ_l (>0) are parameters that make the cost AC_t stay in the steady state, and are respectively defined as $\kappa_m\equiv\frac{\overline{B}_m}{\overline{M}^{\theta}}$ and $\kappa_l\equiv\frac{\overline{B}_l}{\overline{M}^{1-\theta}}$, where $\overline{X}$ stands for the steady state of the variable X_t.

MSZ [2007] take $M_t/B_{m,t}$ and $M_t/B_{l,t}$ as variables in the cost function (named as money transaction costs in their study), instead of using $M_t^{\theta}/B_{m,t}$ and $M_t^{1-\theta}/B_{l,t}$ respectively. In their context, it is assumed that households investing in both middle- and long-term bonds take account of the ratio of the whole amount of money to holdings of each type of bond, individually. However, in practice, such investors would consider the ratio of the whole amount of money to the total amount of illiquid assets, that is, sum of middle- and long-term bond holdings. Consequently, we assume that holdings of middle- and long-term bonds should be respectively compared to a part of the amount of money, not whole amount of money, as described in equation (5).

As for the budget constraint (4), the following points should be noted. First, we specify the cost function AC_t in the household's decision problem, in relation to acquiring the middle- and long-term bonds, as shown above. The cost function has two implications. One is that households perceive both middle- and long-term bonds as riskier assets, entailing a loss of liquidity, relative to one-period bonds. When households invest in middle- and/or long-term bonds, they demand additional money to compensate themselves for the loss of liquidity[6]. In effect, the agents have self-imposed "reserve requirements" (as described in ALSN [2004a, p.14]) of $M_t \times AC_t$ on their middle- and long-term bond investments. The other implication is that this functional form of AC_t

guarantees non-zero demand for these riskier assets in terms of the household budget constraint under the condition that all v_m, v_l, κ_m and κ_l are positive.

Second, we do not assume secondary markets of middle- and long-term bonds. Instead, in each period, the central bank can purchase these bonds as much as households want to sell at prices that are determined through the reverse auction implemented by the central bank. This process implies not only that the reverse auction serves as the secondary market for these bonds but also that the central bank may supply enough money or liquidity for households requirements. Nevertheless, when they invest in middle- and long-term bonds, households save on reserve requirements to provide against unexpected money demand because they regard these bonds as riskier assets[7].

(2) Optimality conditions

According to the above setup, the household seeks to maximize, with respect to a sequence of $\{C_t, N_t, M_t, B_{s,t}, B_{m,t}, B_{l,t}\}$,

$$E_0\sum_{t=0}^{\infty}\beta^t U\left(C_t, C_{t-1}, N_t, \frac{M_t}{P_t}, \frac{M_{t-1}}{P_{t-1}}, e_t\right),$$

subject to budget constraint (4), where the parameter $\beta\in(0, 1)$ is a discount factor.

Defining $M'_t\equiv\frac{M_t}{P_t}$, $B'_{k,t}\equiv\frac{B_{k,t}}{P_t}$ $(k=m, l)$, $\kappa'_m\equiv\frac{\overline{B}'_m}{\overline{M}'^{\theta}}$ and $\kappa'_l\equiv\frac{\overline{B}'_l}{\overline{M}'^{1-\theta}}$, the first-order conditions for the optimizing consumer's problem can be written as follows:

$$\lambda_t=\frac{C_t^{-\sigma}}{C_{t-1}^{h(1-\sigma)}}-\beta h E_t\left(\frac{C_{t+1}^{1-\sigma}}{C_t^{h(1-\sigma)+1}}\right), \tag{6}$$

$$N_t^{\varphi}=\lambda_t\left(\frac{W_t}{P_t}\right), \tag{7}$$

$$\frac{\lambda_t}{P_t}=\beta E_t R_t\left(\frac{\lambda_{t+1}}{P_{t+1}}\right), \tag{8}$$

$$\frac{\lambda_t}{P_t}-\beta E_t H_{m,t}\left(\frac{\lambda_{t+1}}{P_{t+1}}\right)=\left(\frac{\lambda_t}{P_t}\right)\left\{v_m\kappa'_m\left(\frac{M_t'^{\theta}}{B'_{m,t}}\kappa'_s-1\right)\left(\frac{M_t'^{\theta+1}}{B'^2_{m,t}}\right)\right\}, \tag{9}$$

$$\frac{\lambda_t}{P_t}-\beta E_t H_{l,t}\left(\frac{\lambda_{t+1}}{P_{t+1}}\right)=\left(\frac{\lambda_t}{P_t}\right)\left\{v_l\kappa'_l\left(\frac{M_t'^{1-\theta}}{B'_{l,t}}\kappa'_s-1\right)\left(\frac{M_t'^{2-\theta}}{B'^2_{l,t}}\right)\right\}, \tag{10}$$

$$
\begin{aligned}
& e_t^{\theta-1}M'^{-\theta}_t\left(\frac{1}{P_t}\right)-\frac{dc}{2}\left(\frac{1}{P_t}\right)\left[\left\{\exp\left(c\left(\frac{M'_t}{M'_{t-1}}-1\right)\right)-\exp\left(-c\left(\frac{M'_t}{M'_{t-1}}-1\right)\right)\right\}\left(\frac{1}{M'_{t-1}}\right)\right. \\
& \qquad \left.+\beta E_t\left[\left\{\exp\left(-c\left(\frac{M'_{t+1}}{M'_t}-1\right)\right)-\exp\left(c\left(\frac{M'_{t+1}}{M'_t}-1\right)\right)\right\}\left(\frac{M'_{t+1}}{M'^2_t}\right)\right]\right] \\
& -\left(\frac{\lambda_t}{P_t}\right)\left[\frac{v_m}{2}\left(\frac{M'^{\theta}_t}{B'_{m,t}}\kappa'_m-1\right)^2+\frac{v_l}{2}\left(\frac{M'^{1-\theta}_t}{B'_{l,t}}\kappa'_l-1\right)^2\right. \\
& \qquad \left.+\theta v_m\kappa'_m\left(\frac{M'^{\theta}_t}{B'_{m,t}}\kappa'_s-1\right)\left(\frac{M'^{\theta}_t}{B'_{m,t}}\right)+(1-\theta)v_l\kappa'_l\left(\frac{M'^{1-\theta}_t}{B'_{l,t}}\kappa'_l-1\right)\left(\frac{M'^{1-\theta}_t}{B'_{l,t}}\right)\right] \\
& =\frac{\lambda_t}{P_t}-\beta E_t\left(\frac{\lambda_{t+1}}{P_{t+1}}\right),
\end{aligned} \tag{11}
$$

where λ_t represents the Lagrange multiplier for the budget constraint.

The first-order conditions can be interpreted as follows. Equation (6) is the standard expression for the marginal utility of wealth, which, because of habit formation, is based on the marginal utility of consumption - not only the actual in period t but also on the expected in period $t+1$. Equation (7) indicates the standard labour supply schedule. Equations (8) to (10) are the Euler equations for bond holdings at different maturities, which link the marginal utility of wealth across periods. As shown below, these expressions imply a term structure relationship among the nominal yields on one-period, middle- and long-term bonds.

Equation (11) shows the money demand schedule. We should notice roles of both portfolio adjustment costs and costs for purchasing middle- and long-term bonds, that is, the household reserve requirement. First, the presence of portfolio adjustment costs shifts the standard money demand decision from being static to a dynamic one where expectations of future money demand have an effect on contemporaneous money demand[8). Second, the presence of the household reserve requirement causes the relative supply of riskier bonds to matter for money demand decisions. In particular, an increase in the relative amount of these riskier assets raises the demand for money as liquid assets.

(3) Log-linear approximation to the first-order conditions and derivation of the term structure relationship

Below, for any variable X_t except for the real balance M_t' and real bond outstandings $B'_{m,t}$ and $B'_{l,t}$, define

$x_t \equiv \ln(X_t)$,

and, for M_t', $B'_{m,t}$ and $B'_{l,t}$, define

$m_t \equiv \ln(M'_t)$, $b_{m,t} \equiv \ln(B'_{m,t})$ and $b_{l,t} \equiv \ln(B'_{l,t})$,

respectively. Furthermore, with all variables x_t defined as above, define

$\hat{x}_t \equiv x_t - \bar{x}$,

where $\bar{x}$ stands for a steady state of a sequence $\{x_t\}$.

Using the above notations, we can express log-linearized first-order conditions around the steady state as follows:

$$\hat{\lambda}_t=\frac{(\sigma-1)h}{1-\beta h}\hat{c}_{t-1}+\beta\frac{(\sigma-1)h}{1-\beta h}E_t\hat{c}_{t+1}-\frac{\sigma+(\sigma-1)\beta h^2-\beta h}{1-\beta h}\hat{c}_t, \tag{12}$$

$$\varphi\hat{n}_t=\hat{\lambda}_t+\hat{w}_t-\hat{p}_t, \tag{13}$$

$$\hat{\lambda}_t=E_t\hat{\lambda}_{t+1}+\hat{r}_t-E_t\hat{\pi}_{t+1}, \tag{14}$$

$$E_t\hat{\lambda}_{t+1}-\hat{\lambda}_t+\hat{h}_{m,t}-E_t\hat{\pi}_{t+1}=-v_m\left(\frac{\overline{m}}{\overline{b}_m}\right)(\theta\hat{m}_t-\hat{b}_{m,t}), \tag{15}$$

$$E_t\hat{\lambda}_{t+1}-\hat{\lambda}_t+\hat{h}_{l,t}-E_t\hat{\pi}_{t+1}=-v_l\left(\frac{\overline{m}}{\overline{b}_l}\right)\{(1-\theta)\hat{m}_t-\hat{b}_{l,t}\}, \tag{16}$$

$$\begin{aligned}\hat{m}_t=&\frac{\delta_0}{1+\delta_0(1+\beta)}\hat{m}_{t-1}+\beta\frac{\delta_0}{1+\delta_0(1+\beta)}E_t\hat{m}_{t+1}\\&-\left(\frac{1-\delta}{\delta}\right)\frac{1}{1+\delta_0(1+\beta)}\hat{e}_{t-1}-\frac{1}{\delta\{1+\delta_0(1+\beta)\}}\hat{\lambda}_t-\frac{\delta_0}{\delta(\overline{R}-1)\{1+\delta_0(1+\beta)\}}\hat{r}_t\\&-\left(\frac{\overline{R}}{\overline{R}-1}\right)\left[\frac{1}{\delta\{1+\delta_0(1+\beta)\}}\right][\theta v_s(\theta\hat{m}_t-\hat{b}_{m,t})+(1-\theta)v_l\{(1-\theta)\hat{m}_t-\hat{b}_{l,t}\}],\end{aligned} \tag{17}$$

$$\hat{e}_t=\rho_e\hat{e}_{t-1}+\varepsilon_{e,t}, \tag{18}$$

where $\delta_0\equiv\frac{dc^2}{\delta\overline{m}^{1-\delta}}$, $\hat{\pi}_t=\hat{p}_t-\hat{p}_{t-1}$ $(\pi_t\equiv p_t-p_{t-1})$, $\rho_e\in(0,1)$ and $\varepsilon_{e,t}\sim i.i.d.(0,\sigma_e^2)$.

As mentioned above, equations (14) to (16) are the Euler equations for bond holdings at different maturities, and imply the term structure relationship among their yields. Combining equation (14) with equation (15) or (16) yields the following equations, respectively.

$$\hat{h}_{m,t}=\hat{r}_t+v_m\left(\frac{\overline{m}}{\overline{b}_m}\right)(\theta\hat{m}_t-\hat{b}_{m,t}), \tag{19}$$

$$\hat{h}_{l,t}=\hat{r}_t+v_l\left(\frac{\overline{m}}{\overline{b}_l}\right)\{(1-\theta)\hat{m}_t-\hat{b}_{l,t}\}. \tag{20}$$

These equations demonstrate the term structure of interest rates in terms of the relationship between the one-period nominal interest rate and the one-period nominal holding return on middle- or long-term bond, which is shifted by the modified ratio of money to each bond holding. Accordingly, expressions (19) and (20) capture an essential feature of Tobin's [1969] framework - that spreads between interest rates should reflect the relative quantities of assets.

As the aim of this study is to examine monetary policy effects of purchasing government bonds on the term structure of interest rates, we will illustrate some measures to indicate the effect. The effect is measured by changes of level, slope, and curvature of the yield curve. For a zero-coupon bond with a term to maturity of n at period t, log-linear approximation of one-period nominal holding returns in period t, $h_t^{(n)}$, is expressed as

$$h_t^{(n)}=nr_t^{(n)}-(n-1)E_t r_{t+1}^{(n-1)}, \tag{21}$$

where $r_t^{(n)}$ denotes net nominal interest rates of the bond. Meanwhile, we define level, slope, and curvature of the yield curve as follows, according to Ang and Piazzesi [2003] and Bianchi, Mumtaz and Surico [2009]:

$$\text{Level } (L_t) = \frac{1}{3}(r_t^{(n_1)}+r_t^{(n_2)}+r_t^{(n_3)}),$$

$$\text{Slope } (S_t) = r_t^{(n_3)}-r_t^{(n_1)}, \tag{22}$$

$$\text{Curvature } (V_t) = r_t^{(n_2)}-\frac{1}{2}(r_t^{(n_1)}+r_t^{(n_3)}),$$

where n_1, n_2 and n_3 are residual maturities with $n_1<n_2<n_3$.

Combining the above definitions with equation (21) and using the notation of one-period holding returns[9],

$$\widehat{L}_t\approx\frac{1}{3}(\widehat{r}_t+\widehat{h}_{m,t}+\widehat{h}_{l,t}),$$

$$\widehat{S}_t\approx\widehat{h}_{l,t}-\widehat{r}_t, \tag{23}$$

$$\widehat{V}_t\approx\widehat{h}_{m,t}-\frac{1}{2}(\widehat{r}_t+\widehat{h}_{l,t}).$$

Finally, according to equations (19) and (20), the term premia on middle- and long-term yields are respectively given by

$$\widehat{TP}_{m,t}\equiv\widehat{h}_{m,t}-\widehat{r}_t=v_m\left(\frac{\overline{m}}{\overline{b}_m}\right)(\theta\widehat{m}_t-\widehat{b}_{m,t}), \tag{19'}$$

$$\widehat{TP}_{l,t}\equiv\widehat{h}_{l,t}-\widehat{r}_t=v_l\left(\frac{\overline{m}}{\overline{b}_l}\right)\{(1-\theta)\widehat{m}_t-\widehat{b}_{l,t}\}. \tag{20'}$$

2. Firms

(1) Aggregate price dynamics and optimal price setting

As stated above, we assume a continuum of firms indexed by $j\in[0, 1]$. Each firm produces a differentiated good $Y_t(j)$ in period t, but all firms use an identical technology, represented by the following production function:

$$Y_t(j)=A_t N_t(j)^{1-\alpha}, \tag{24}$$

where we presuppose no capital accumulation in the economy[10]. A_t denotes the level of technology in period t, assumed to evolve exogenously over time as follows:

$$a_t=\rho_a a_{t-1}+\varepsilon_{a,t}, \tag{25}$$

where $a_t\equiv\ln A_t$, $\rho_a\in(0, 1)$ and $\varepsilon_{a,t}\sim i.i.d.(0, \sigma_a^2)$ which indicates a shock to the technology. $N_t(j)$ is the number of work-hours hired from households by firm j in period t, and $\alpha\in[0, 1]$

represents the share of capital in production.

All firms face an identical isoelastic demand schedule given by the following, and take the aggregate price level P_t in period t and aggregate consumption index C_t in period t as given:

$$Y_t(j)=\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}C_t, \tag{26}$$

where $P_t(j)$ is the price of good j in period t, ε stands for constant price elasticity, and

$$P_t\equiv\left[\int_0^1 P_t(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}. \tag{27}$$

Here, consider the Calvo [1986] model of staggered price setting with the following modification. In the period between price reoptimizations, firms mechanically adjust their prices according to some indexation rule, named as "lagged inflation indexation" by Christiano, Eichenbanm and Evans [2005]. Formally, a firm that has an opportunity to reoptimize its price in period t with probability $1-\eta$, sets an optimal price $P_t{}^*$ in that period. In subsequent periods (i.e., until it has an opportunity to reoptimize prices again), its price is adjusted according to the following rules of partial indexation to past inflation,

$$P_{t+k|t}=P_{t+k-1|t}(\Pi_{t+k-1})^{\omega} \tag{28}$$

for $k=1, 2, \cdots$, and

$$P_{t,t}=P_t^*, \tag{29}$$

where $P_{t+k|t}$ denotes the price effective in period $t+k$ for the firm that last reoptimized its price in period t, $\Pi_t\equiv\frac{P_t}{P_{t-1}}$ is the aggregate gross inflation rate in period t, and $\omega\in[0, 1]$ is a parameter measuring the degree of indexation.

Combining the definition of aggregate price (27) with the firm's price-adjusting rules (28) and (29), the aggregate price dynamics are described as

$$\Pi_t^{1-\varepsilon}=\eta(\Pi_{t-1}^{\omega})^{1-\varepsilon}+(1-\eta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}. \tag{30}$$

A log-linear approximation to the aggregate price dynamics around a steady state with zero inflation yields

$$\hat{\pi}_t-\eta\omega\hat{\pi}_{t-1}=(1-\eta)(\hat{p}_t^*-\hat{p}_{t-1}), \tag{31}$$

where, as in the section on households, lowercase letters denote the natural logs of the corresponding variables and the hat symbol (^) indicates deviations from the steady-state values.

Next, we will derive a firm's optimal price setting. A firm reoptimizing in period t will choose the price $P_t{}^*$ that maximizes the current market value of the profits generated, subject to a sequence of demand constraints and the rule of price adjustment described above. Formally,

$$\max_{P_t^*}\sum_{k=0}^{\infty}\eta^k E_t[Q_{t,t+k}\{P_{t+k|t}Y_{t+k|t}-\Psi_{t+k}(Y_{t+k|t})\}] \tag{32}$$

subject to $Y_{t+k|t}=\left(\frac{P_{t+k|t}}{P_t}\right)^{-\varepsilon} C_{t+k}$ (33)

and equation (28) for k = 0, 1, 2, ⋯. In the above equations $Y_{t+k|t}$ denotes the output in period t + k of a firm that last reoptimized its price in period t, $Q_{t,t+k}\equiv\beta^k\left(\frac{\lambda_{t+k}}{\lambda_t}\right)^{-\sigma}$ is the stochastic discount factor for nominal payoff, and $\Psi(\bullet)$ is the cost function.

From the first-order conditions with respect to the above problem, the following equation can be derived:

$$\sum_{k=0}^{\infty}\eta^k E_t\left[Q_{t,t+k}Y_{t+k|t}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}\left\{\frac{P_t^*}{P_{t-1}}-\upsilon\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{-\omega}P_{t+k}MC_{t+k|t}\right\}\right]=0, \tag{34}$$

where $MC_{t+k|t}\equiv\frac{\Psi'_{t+k}(Y_{t+k|t})}{P_{t+k}}\left(=\left(\frac{W_{t+k}}{P_{t+k}}\right)\left(\frac{\partial Y_{t+k|t}}{\partial N_t(j)}\right)^{-1}\right)$ is the real marginal cost in period t + k for a firm whose price was last set in period t, and $\upsilon\equiv\frac{\varepsilon}{\varepsilon-1}$ represents the desired or frictionless markup. Log-linearizing equation (34) around the steady state leads to the following optimal price-setting equation:

$$\hat{p}_t^*-\hat{p}_t=(1-\beta\eta)\sum_{k=0}^{\infty}(\beta\eta)^k E_t[\hat{mc}_{t+k|t}+(\hat{p}_{t+k}-\hat{p}_t)-\omega(\hat{p}_{t+k-1}-\hat{p}_{t-1})]. \tag{35}$$

(2) Derivation of the hybrid New-Keynesian Phillips Curve (NKPC)

Because we presuppose no capital accumulation in the economy, market clearing of goods market requires $Y_t(j)$ = $C_t(j)$ for all j and all t. Letting aggregate output be defined as $Y_t\equiv\left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, combined with equation (1) and the goods market clearing condition, yields

$$Y_t=C_t. \tag{36}$$

Meanwhile, clearing labour in the labour market requires

$$N_t=\int_0^1 N_t(j)dj. \tag{37}$$

From equations (24), (26), (36) and (37), we can derive the following approximate aggregated production function:

$$y_t=a_t+(1-\alpha)n_t. \tag{38}$$

By equation (38), an individual firm's marginal cost in terms of the economy's average real marginal cost is defined as

$$mc_t=w_t-p_t-\frac{1}{1-\alpha}(a_t-\alpha y_t)-\ln(1-\alpha). \tag{39}$$

This equation implies

$$mc_{t+k|t}=(w_{t+k}-p_{t+k})-\frac{1}{1-\alpha}(a_{t+k}-\alpha y_{t+k|t})-\ln(1-\alpha). \tag{40}$$

Then,

$$\begin{aligned}\hat{mc}_{t+k|t} &= \hat{mc}_{t+k} + \frac{\alpha}{1-\alpha}(\hat{y}_{t+k|t} - \hat{y}_{t+k}) \\ &= \hat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\{(\hat{p}^*_t - \hat{p}_{t+k}) + \omega(\hat{p}_{t+k-1} - \hat{p}_{t-1})\}\,,\end{aligned} \tag{41}$$

where the second equality follows from the demand schedule (33), combined with the goods market clearing condition (36).

Substituting equation (41) into equation (35), and using equation (31), the hybrid New-Keynesian Phillips Curve (NKPC) is described as follows:

$$\hat{\pi}_t = \frac{\omega}{1+\omega\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\omega\beta}E_t\hat{\pi}_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\omega\beta)}\Theta\hat{mc}_t, \tag{42}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1.$

Finally, we will express the equation of $\hat{mc}_t$. Eliminating $w_t - p_t$ in equation (39) by using equation (13) yields

$$\hat{mc}_t = -\hat{\lambda}_t + \varphi\hat{n}_t - \frac{1}{1-\alpha}(\hat{a}_t - \alpha\hat{y}_t). \tag{43}$$

Equation (38) is rewritten by

$$\hat{n}_t = \frac{1}{1-\alpha}(\hat{y}_t - \hat{a}_t), \tag{44}$$

so that substituting equation (44) into equation (43), and rearranging the resulting equation,

$$\hat{mc}_t = -\hat{\lambda}_t + \frac{\varphi+\alpha}{1-\alpha}\hat{y}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t. \tag{45}$$

Eliminating $\hat{\lambda}_t$ in the above equation by using equation (12) combined with $\hat{y}_t = \hat{c}_t$, and rearranging the resultant equation, we obtain

$$\hat{mc}_t = \left\{\frac{\varphi+\alpha}{1-\alpha} + \frac{\sigma+(\sigma-1)\beta h^2 - \beta h}{1-\beta h}\right\}\hat{y}_t - \frac{(\sigma-1)h}{1-\beta h}\hat{y}_{t-1} - \beta\frac{(\sigma-1)h}{1-\beta h}E_t\hat{y}_{t+1} - \frac{\varphi+\alpha}{1-\alpha}\hat{a}_t. \tag{46}$$

In this equation, by equation (25),

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{a,t}, \tag{47}$$

where $\rho_a \in (0, 1)$ and $\varepsilon_{a,t} \sim i.i.d.(0, \sigma_a^2)$.

3. The central bank

We assume that the central bank sets the one-period nominal interest rate as a policy rate according to an augmented Taylor-type interest rate rule. It is also presumed that the interest rate rule could be affected by the supply of high-powered money, as described in Taylor [2001].

The central bank supplies high-powered money as much as households demand. As explained above, the household saves reserve requirements when it invests in illiquid assets, so that demand for high-powered money is presumed to consist of households' holdings of illiquid assets. Accordingly we define the demand function of high-powered money as follows:

$$HM_t^D=\kappa_{cb}B_{m,t}^{\theta_{cb}}B_{l,t}^{1-\theta_{cb}}, \tag{48}$$

where HM_t^D denotes the nominal demand for high-powered money in period t, $\theta_{cb}\in[0,1]$ is a parameter; κ_{cb} is a parameter that makes the demand for high-powered moneystay in the steady state, and is defined as $\kappa_{cb}\equiv\frac{\overline{HM^D}}{\overline{B}_m^{\theta_{cb}}\overline{B}_l^{1-\theta_{cb}}}$. Equation (48) is consistent with the view that the household saves on reserve requirements in the cost function (5), that is, $\frac{M_t^{\theta}}{B_{m,t}}$ and $\frac{M_t^{1-\theta}}{B_{l,t}}$.

On the other hand, the central bank supplies high-powered money in response to its demand as follows:

$$HM_t^S=\exp(\varsigma_t)HM_t^D, \tag{49}$$

where HM_t^S denotes the nominal supply of high-powered money in period t, and ζ_t follows the process below.

$$\varsigma_t=\rho_\varsigma\varsigma_{t-1}+\varepsilon_{\varsigma,t}, \tag{50}$$

where $\rho_\zeta\in(0,1)$ and $\varepsilon_{\varsigma,t}\sim i.i.d.(0,\sigma_\varsigma^2)$. In equation (49), the role of the term $\exp(\zeta_t)$ is to adjust the supply of high-powered money according to its demand, so that increases in ζ_t imply a monetary easing policy in the sense that the central bank supplies more high-powered money to meet its demand than in previous periods.

Substituting equation (48) into equation (49),

$$HM_t^S=\kappa_{cb}\exp(\varsigma_t)B_{m,t}^{\theta_{cb}}B_{l,t}^{1-\theta_{cb}}. \tag{51}$$

This functional form implies that the central bank supplies high-powered money by purchasing illiquid assets, that is, middle- and long-term government bonds, from the agents[11].

In the following paragraphs, we consider two types of interest rate rules according to a way to incorporate the supply of high-powered money into the rule, so that henceforth, HM_t^S is expressed as HM_t.

(1) Policy rule 1 regime

The gross one-period nominal interest rate in period t, R_t, responds not only to the rate in the previous period, and to deviations of output and inflation rate from their steady-state values, but also to the ratio of the demand for money to the supply of high-powered money. Formally,

$$\ln(R_t/\overline{R})=\rho_{r1}\ln(R_{t-1}/\overline{R})+(1-\rho_{r1})\{\rho_{\pi1}\ln(\Pi_t/\overline{\Pi})+\rho_{y1}\ln(Y_t/\overline{Y})+\rho_\mu\ln(\mu_t/\overline{\mu})\}, \tag{52}$$

where $\rho_{r1} \in (0, 1)$, $\rho_{\pi 1} > 1$ (for fulfilling the Taylor principle), $\rho_{y1} > 0$, $\rho_{\mu} > 0$, $\mu_t \equiv \frac{M_t}{HM_t} = \frac{M_t}{\kappa_{cb} \exp(\varsigma_t) B_{m,t}^{\theta_{cb}} B_{l,t}^{1-\theta_{cb}}}$ and $\bar{\mu} = \frac{\bar{M}}{\overline{HM}}$. Log-linearizing equation (52) around the steady state, this rule is rewritten as

$$\hat{r}_t = \rho_{r1} \hat{r}_{t-1} + (1-\rho_{r1}) [\rho_{\pi 1} \hat{\pi}_t + \rho_{y1} \hat{y}_t + \rho_{\mu} \{\widehat{m}_t - \theta_{cb} \hat{b}_{m,t} - (1-\theta_{cb}) \hat{b}_{l,t} - \varsigma_t\}] . \tag{53}$$

(2) Policy rule 2 regime

The central bank sets a desired nominal rate R_t^* as

$$\ln (R_t^* / \bar{R}) = \rho_{r2} \ln (R_{t-1} / \bar{R}) + (1-\rho_{r2}) \{\rho_{\pi 2} \ln (\Pi_t / \bar{\Pi}) + \rho_{y1} \ln (Y_t / \bar{Y})\}, \tag{54}$$

where $\rho_{r2} \in (0, 1)$, $\rho_{\pi 2} > 1$ (for fulfilling the Taylor principle), and $\rho_{y2} > 0$. This equation yields, by log-linear approximation,

$$\hat{r}_t^* = \rho_{r2} \hat{r}_{t-1} + (1-\rho_{r2}) (\rho_{\pi 2} \hat{\pi}_t + \rho_{y2} \hat{y}_t). \tag{55}$$

Here, we consider a relationship among the one-period rate r_t, the desired one r_t^* set by equation (55), and the supply of high-powered money HM_t, according to the argument by Taylor [2001]. For a constant $\alpha > 0$, equation (3) in Taylor (p.43) is expressed as

$$\ln HM_t = \ln HM_{t-1} + \alpha (r_{t-1} - r_{t-1}^*), \tag{56}$$

which is the supply function of high-powered money. This equation suggests that if $r_{t-1} > r_{t-1}^*$, then the central bank would increase the supply of high-powered money in period t in order to lower the one-period nominal rate in period t, and vice versa. The argument implies the following relationship between $r_{t-1} - r_{t-1}^*$ and $\ln HM_t$ - $\ln HM_{t-1}$:

$$r_t - r_t^* = -\chi (\ln HM_t - \ln HM_{t-1}), \tag{57}$$

where $\chi\,(> 0)$ is a parameter that indicates an adjustment factor. From a technical viewpoint, we add the following constraint to $\ln HM_{t-1}$ in equation (57):

$$r_t - r_t^* = -\chi (\ln HM_t - \rho_{\varsigma} \ln HM_{t-1}). \tag{58}$$

Next, substituting equation (51) into equation (58),

$$r_t - r_t^* = -\chi (\ln \kappa_{cb} \exp (\varsigma_t) B_{m,t}^{\theta_{cb}} B_{l,t}^{1-\theta_{cb}} - \rho_{\varsigma} \ln \kappa_{cb} \exp (\varsigma_{t-1}) B_{m,t-1}^{\theta_{cb}} B_{l,t-1}^{1-\theta_{cb}}),$$

which is rewritten, by using equation (50), as

$$\hat{r}_t = \hat{r}_t^* - \chi \{\theta_{cb} (\hat{b}_{m,t} - \rho_{\varsigma} \hat{b}_{m,t-1}) + (1-\theta_{cb}) (\hat{b}_{l,t} - \rho_{\varsigma} \hat{b}_{l,t-1}) + \hat{\pi}_t + \varepsilon_{\varsigma,t}\}. \tag{59}$$

Combing equations (55) and (59), the other rule is specified by

$$\hat{r}_t = \rho_{r2} \hat{r}_{t-1} + (1-\rho_{r2}) (\rho_{\pi 2} \hat{\pi}_t + \rho_{y2} \hat{y}_t)$$
$$- \chi \{\theta_{cb} (\hat{b}_{m,t} - \rho_{\varsigma} \hat{b}_{m,t-1}) + (1-\theta_{cb}) (\hat{b}_{l,t} - \rho_{\varsigma} \hat{b}_{l,t-1}) + \hat{\pi}_t + \varepsilon_{\varsigma,t}\}. \tag{60}$$

4. The government

Fiscal deficits, defined as transfer payments plus redemption of past issued bonds minus tax and seignorage revenues, are financed by issuing one-period, middle- and long-term bonds. It is assumed that these bonds are zero-coupon bonds issued at the nominal prices of $Q_{s,t}$ for the one-period, $Q_{m,t}$ for the middle-term, and $Q_{l,t}$for the long-term bond; and each bond is redeemed for one unit of money in period $t+1$, $t+m$ and $t+l$ $(1 < m < l)$ respectively. Then, the government budget constraint is formally given by

$$\frac{1}{P_t}\{(M_t+Q_{s,t}B_{s,t}+Q_{m,t}B_{m,t}+Q_{l,t}B_{l,t})-(M_{t-1}+B_{s,t-1}+B_{m,t-m}+B_{l,t-l})\}$$
$$=\frac{1}{P_t}(TR_t-TA_t). \tag{61}$$

Define $TR'_t\equiv\frac{TR_t}{P_t}$, $tr_t\equiv\ln TR'_t$, $TA'_t\equiv\frac{TA_t}{P_t}$ and $ta_t\equiv\ln TA'_t$. Furthermore, denoting $R_{k,t}$ (k=m and l) as the gross nominal interest rates of bonds with residual maturities of m and l, respectively, $Q_{k,t}$ is expressed by

$$Q_{k,t}=\frac{1}{R^k_{k,t}}. \tag{62}$$

Using the above definitions and equation (62), log-linearized budget constraint (61) is

$$\overline{M}\widehat{m}_t+\frac{\overline{B}_s}{\overline{R}}(\hat{b}_{s,t}-\hat{r}_t)+\frac{\overline{B}_m}{\overline{R}^m_m}(\hat{b}_{m,t}-\hat{r}_{m,t})+\frac{\overline{B}_l}{\overline{R}^l_l}(\hat{b}_{l,t}-\hat{r}_{l,t})-\Big\{\frac{\overline{M}}{\overline{\Pi}}(\widehat{m}_{t-1}-\hat{\pi}_t)$$
$$+\frac{\overline{B}_s}{\overline{\Pi}}(\hat{b}_{s,t-1}-\hat{\pi}_t)+\frac{\overline{B}_m}{\overline{\Pi}^m}(\hat{b}_{m,t-m}-\sum_{i=0}^{m-1}\hat{\pi}_{t-i})+\frac{\overline{B}_l}{\overline{\Pi}^l}(\hat{b}_{l,t-l}-\sum_{i=0}^{l-1}\hat{\pi}_{t-i})\Big\}$$
$$=\hat{tr}_t-\hat{ta}_t, \tag{63}$$

where the relationship of $Q_{s,t}=\frac{1}{R_t}$ is used.

Here, on the assumption that the interest rate on each of the middle- and long-term bonds is approximated by the one-period holding return on the corresponding bond according to the argument in Appendix A and with the middle- and long-term interest rates in equation (63) replaced with the corresponding holding return, the variables $\{\widehat{m}_t\}$, $\{\hat{r}_t\}$, $\{\hat{h}_{m,t}\}$, $\{\hat{h}_{l,t}\}$ and $\{\hat{\pi}_t\}$ in equation (63) are determined by the behaviour of households, firms and the central bank, as explained above. Then, on the assumption that the government exogenously decides transfer payments and tax, the issuing processes of middle- and long-term bonds are important to complete the model structure, because the issue of the one-period bond is determined as a residual means of public finance, following the budget constraint. For simplicity, we assume that the issue of middle- and long-term bonds follows a simple AR(1) process,

$$B'_{k,t}=\exp(\varepsilon_{bk,t})(B'_{k,t-1})^{\rho_{bk}}, \tag{64}$$

where $\rho_{bk}\in(0,1)$ and $\varepsilon_{bk,t}\sim i.i.d.(0,\sigma^2_{bk})$ for $k=m$ and l. Equation (64) is log-linearized as

$$\hat{b}_{k,t}=\rho_{bk}\hat{b}_{k,t-1}+\varepsilon_{bk,t} \tag{65}$$

for k=m and l.

5. Complete model

We have described the behaviour of agents in the economy. To complete the model, the equilibrium condition of C_t = Y_t must be applied to households' behaviour, same as in firms' behaviour. The complete model is presented in Appendix B.

Ⅲ Bayesian estimation of the model

1. Preliminary setting

We use monthly data of the U.S. for the period January 2000 - October 2009 and of the U.K. for the period January 2008 - January 2010. The end point for U.S. data is determined by the fact that the FRB completed the special program (longer-dated Treasury purchase program) at the end of October 2009. With the U.K., the data series begins when the BOE started outright purchases of conventional gilts as a regular open market operation and ends in the month when purchases under the APF was suspended.

Data needed to construct the model are as follows: output, consumer goods price, money supply, interest rates (one-period, middle- and long-term), outstandings of government bonds (middle-term and long-term). As output and real balances have a clear time trend, we use the detrended series of these variables for estimation. Data series and their sources are listed in Appendix C.

We do not estimate all parameters in the model, but a part of parameters are set according to previous studies. These parameters and their values are shown in Table 1.

As the basic structure of the model is similar to that of ALSN [2004a, b], we follow their estimation results for the following parameters:

σ, β, h, δ_0, δ, $\rho_{r1}(=\rho_{r2})$, $\rho_{\pi 1}$, ρ_{y1} $(=\rho_{y2})$, ρ_μ, ρ_e and ρ_a for the U.S.;

β, h, δ_0, $\rho_{r1}(=\rho_{r2})$, $\rho_{\pi 1}$, ρ_{y1} $(=\rho_{y2})$, ρ_μ, $\rho_e$and ρ_a for the U.K..

Because ALSN [2004b] do not estimate parameters of σ and δ for the U.K., we set the same values for the parameters as in the U.S.; that is, $\sigma=2$ and $\delta=5$. The set of $\sigma=2$ is consistent with Fukac, Pagan and Pavlov [2006] and DiCecio and Nelson [2009].

ALSN [2004a, b] do not show estimates of α and φ but estimate the value of $(\varphi+\alpha)/(1-\alpha)$[12]. We set α at 0.36 for both of countries, following MSZ [2007] for the U.S., and DiCecio and Nelson [2007] for the U.K. Furthermore, φ is set at 1 for the U.S. and 2 for the U.K., according to MSZ [2007] and DiCecio and Nelson [2009], respectively. Finally, $\rho_{\pi 2}$ is set at 3 for both countries.

Table 1. Calibrated Parameters

Parameters	US	UK
α	2	2
β	0.997	0.997
h	0.95	0.95
φ	1	2
δ	5	5
δ_0	4	4
ρ_e	0.98	0.99
α	0.36	0.36
ρ_a	0.96	0.95
ρ_{r1}, ρ_{r2}	0.8	0.8
$\rho_{\pi 1}$	2	1.9
$\rho_{\pi 2}$	3	3
ρ_{y1}, ρ_{y2}	0.4	0.3
ρ_u	1	0.3

2. Parameters estimation

The remaining parameters - v_m, v_l, θ, θ_{cb}, χ, ρ_ζ, ω, η, Θ, ρ_{bm} and ρ_{bl} - should be estimated. In particular, parameters v_m, v_l, θ, θ_{cB}, χ, and ρ_ζ are related to an important concept introduced into the model. Estimates of these parameters must meet the following conditions:

$v_m>0$, $v_l>0$, $\chi>0$, θ and $\theta_{cb} \in [0, 1]$, and $\rho_\zeta \in (0, 1)$. (66)

To estimate unknown parameters, we conduct Bayesian inference using a Markov Chain Monte Carlo (MCMC) method, which is now a standard technique for estimating the DSGE model[13). Estimation results are presented in Table 2 for the U.S. and in Table 3 for the U.K. With respect to all parameters, except for standard deviation on disturbances, estimated values fulfil parameter conditions including (66)[14). It is noticed that estimates of all parameters are very similar between cases in the U.S. and U.K. In particular, estimates of ω, η, Θ, ρ_{bm} and ρ_{bl} are exceedingly close among all cases, that is, not only between both countries but also between policy rule regimes in either country. For that reason, we will start with illustrating an interpretation of estimation results on these parameters. Here, a tilde over a parameter (e.g. $\tilde{\alpha}$) denotes an estimate of the parameter.

First, both $\tilde{\rho}_{bm}$ and $\tilde{\rho}_{bl}$ are around 0.7 for all cases. These estimates are very similar to those set by MSZ [2007] - ρ_{bm} = 0.73 and ρ_{bl} = 0.75, respectively, for the U.S.

Second, for all cases, $\tilde{\eta}$ is around 0.9, $\tilde{\omega}$ around 0.6, and $\tilde{\Theta}$ around 0.2. As the parameter η represents the probability that a firm does not have an opportunity to reoptimize its price, high values of $\tilde{\eta}$ imply that price changes in consumer goods are significantly sluggish, which is consistent with the lower growth of CPI in the 2000s[15].

Based on the premise of β =0.997, the slope of NKPC, $\frac{(1-\tilde{\eta})(1-\beta\tilde{\eta})}{\tilde{\eta}(1+\tilde{\omega}\beta)}\tilde{\Theta}$, is 3.09×10^{-4} (in the policy rule 1 regime) ~ 1.10×10^{-3} (in the policy rule 2 regime) for the U.S., and 1.90×10^{-3} ~ 1.17×10^{-3} for the U.K. Any estimates of the slope is remarkably low compared with estimates by ALSN [2004a, b], but it is close to the estimate by DiCecio and Nelson [2007][16]. On the other hand, the coefficient on $\widehat{\pi}_{t-1}$ in NKPC, $\frac{\tilde{\omega}}{1+\tilde{\omega}\beta}$, is estimated as 0.375 for all cases. This value is much larger than that estimated on the slope of NKPC, which suggests strong inflation stickiness in both countries.

Next, we will interpret estimates of the important parameters in the model, that is, v_m, v_l, θ, θ_{cb}, ρ_ς and χ.

(1) v_m, v_l and θ

Estimates of v_m, v_l and θ have the following implications for the U.S. and U.K. First, changes in real outstandings of middle- and long-term government bonds influence the real balance.

Second, the relative weight of the real balance in terms of real outstandings of middle- or long-term bond affects one-period nominal holding returns on the corresponding bond. Furthermore, from equations (19') and (20'), the partial impact that a 1% increase in the real balance makes on term premia of one-period nominal holding returns on middle- and long-term bonds is respectively measured by $\frac{\partial}{\partial\widehat{m}_t}(\widehat{TP}_{k,t})=\tilde{v}_k\left(\frac{\bar{m}}{\bar{b}_k}\right)\tilde{\theta}$ for k = m and l. The partial effect of the real outstandings of these bonds is also measured by $\frac{\partial}{\partial\widehat{b}_{k,t}}(\widehat{TP}_{k,t})=-\tilde{v}_k\left(\frac{\bar{m}}{\bar{b}_k}\right)$ for k = m and l. For the U.S., as $\bar{m}$ = 8.098, $\bar{b}_s$ = 7.206 and $\bar{b}_l$ = 6.642 in our sample period, $\frac{\partial}{\partial\widehat{m}_t}(\widehat{TP}_{m,t})$ = 0.2238, $\frac{\partial}{\partial\widehat{b}_{m,t}}(\widehat{TP}_{m,t})$ = -0.4459, $\frac{\partial}{\partial\widehat{m}_t}(\widehat{TP}_{l,t})$ = 0.2785 and $\frac{\partial}{\partial\widehat{b}_{l,t}}(\widehat{TP}_{l,t})$ = -0.5048 in the policy rule 1 regime; the corresponding values in the policy rule 2 regime are 0.2785, -0.5418, 0.2583 and -0.5023, respectively. With the U.K., as $\bar{m}$ = 7.457, $\bar{b}_s$ = 11.90 and $\bar{b}_l$ = 12.46 in our sample period, $\frac{\partial}{\partial\widehat{m}_t}(\widehat{TP}_{m,t})$ = 0.1437, $\frac{\partial}{\partial\widehat{b}_{m,t}}(\widehat{TP}_{m,t})$ = -0.2939, $\frac{\partial}{\partial\widehat{m}_t}(\widehat{TP}_{l,t})$ = 0.2049 and $\frac{\partial}{\partial\widehat{b}_{l,t}}(\widehat{TP}_{l,t})$ = -0.4190 in the policy rule 1 regime; the corresponding values in the policy rule 2 regime are 0.2536, -0.5031, 0.1675 and -0.3322, respectively.

These measures indicate a common feature for both countries: changes in real balances and outstandings of government bonds have larger impacts on nominal holding returns on long-term bonds than on those of middle-term bonds in the policy rule 1 regime, but in the policy rule 2 regime changes in these variables have larger effects on nominal holding returns on middle-term bonds.

Table 2. Estimation Results on US Parameters

1. Policy Rule 1 Regime

Parameters	Prior			Posterior	
	Distribution	Mean	S. D.	Mean	90% interval
v_m	gamma	0.400	0.0300	0.3968	[0.3566, 0.4364]
v_l	gamma	0.500	0.0500	0.4140	[0.3684, 0.4485]
θ	beta	0.500	0.0100	0.5020	[0.4857, 0.5175]
θ_{cb}	beta	0.500	0.0100	0.5120	[0.4967, 0.5259]
ρ_ς	beta	0.750	0.1000	0.6892	[0.6483, 0.7397]
η	beta	0.500	0.1000	0.9522	[0.9513, 0.9529]
ω	gamma	0.600	0.0010	0.6000	[0.5984, 0.6017]
Θ	beta	0.200	0.0050	0.1943	[0.1866, 0.2021]
ρ_{bm}	beta	0.700	0.0100	0.7262	[0.7075, 0.7430]
ρ_{bl}	beta	0.700	0.0100	0.7054	[0.6907, 0.7193]
σ_e	inv. gamma	0.300	0.0500	0.2819	[0.2443, 0.3170]
σ_a	inv. gamma	0.100	0.0200	0.1364	[0.1229, 0.1505]
σ_{bm}	inv. gamma	0.100	0.0500	0.0704	[0.0592, 0.0804]
σ_{bl}	inv. gamma	0.100	0.0500	0.0938	[0.0808, 0.1058]
σ_ς	inv. gamma	0.010	0.1000	0.0251	[0.0228, 0.0280]

2. Policy Rule 2 Regime

Parameters	Prior			Posterior	
	Distribution	Mean	S. D.	Mean	90% interval
v_m	gamma	0.400	0.0300	0.4821	[0.4312, 0.5343]
v_l	gamma	0.500	0.0500	0.4120	[0.3578, 0.4699]
θ	beta	0.500	0.0100	0.5141	[0.4945, 0.5281]
θ_{cb}	beta	0.500	0.0100	0.4873	[0.4731, 0.5009]
ρ_ς	beta	0.750	0.1000	0.9613	[0.9385, 0.9808]
η	beta	0.500	0.1000	0.9117	[0.8999, 0.9251]
ω	gamma	0.600	0.0010	0.5999	[0.5983, 0.6016]
Θ	beta	0.200	0.0050	0.1996	[0.1913, 0.2068]
ρ_{bm}	beta	0.700	0.0100	0.7339	[0.7186, 0.7483]
ρ_{bl}	beta	0.700	0.0100	0.6961	[0.6805, 0.7120]
χ	gamma	1.000	0.0100	0.9908	[0.9742, 1.0074]
σ_e	inv. gamma	1.500	0.1000	0.9334	[0.8482, 1.0030]
σ_a	inv. gamma	0.050	0.0020	0.0490	[0.0435, 0.0542]
σ_{bm}	inv. gamma	0.050	0.0050	0.0601	[0.0538, 0.0657]
σ_{bl}	inv. gamma	0.100	0.0100	0.0949	[0.0853, 0.1054]
σ_ς	inv. gamma	0.100	0.0050	0.0675	[0.0613, 0.0724]

Table 3. Estimation Results on UK Parameters

1. Policy Rule 1 Regime

Parameters	Prior			Posterior	
	Distribution	Mean	S. D.	Mean	90% interval
v_m	gamma	0.800	0.1000	0.4689	[0.3706, 0.5560]
v_l	gamma	0.600	0.0500	0.7002	[0.6214, 0.7883]
θ	beta	0.500	0.0100	0.4890	[0.4740, 0.5069]
θ_{cb}	beta	0.500	0.0100	0.5070	[0.4918, 0.5246]
ρ_ς	beta	0.900	0.0200	0.8588	[0.8193, 0.9022]
η	beta	0.900	0.0100	0.8856	[0.8719, 0.8994]
ω	gamma	0.600	0.0010	0.6000	[0.5986, 0.6016]
Θ	beta	0.200	0.0050	0.2009	[0.1931, 0.2092]
ρ_{bm}	beta	0.700	0.0100	0.7058	[0.6911, 0.7232]
ρ_{bl}	beta	0.700	0.0100	0.6971	[0.6798, 0.7134]
σ_e	inv. gamma	0.800	0.5000	2.7822	[1.8045, 3.6957]
σ_a	inv. gamma	0.100	0.0100	0.1349	[0.1077, 0.1643]
σ_{bm}	inv. gamma	0.100	0.0050	0.1077	[0.0934, 0.1205]
σ_{bl}	inv. gamma	0.100	0.0100	0.1383	[0.1107, 0.1644]
σ_ς	inv. gamma	0.010	0.0200	0.0828	[0.0564, 0.1060]

2. Policy Rule 2 Regime

Parameters	Prior			Posterior	
	Distribution	Mean	S. D.	Mean	90% interval
v_m	gamma	0.800	0.1000	0.8025	[0.6645, 0.9463]
v_l	gamma	0.600	0.0500	0.5552	[0.4788, 0.6314]
θ	beta	0.500	0.0100	0.5041	[0.4883, 0.5200]
θ_{cb}	beta	0.500	0.0100	0.4930	[0.4779, 0.5092]
ρ_ς	beta	0.900	0.0200	0.8505	[0.8246, 0.8757]
η	beta	0.900	0.0100	0.8944	[0.8828, 0.9067]
ω	gamma	0.600	0.0010	0.6000	[0.5983, 0.6015]
Θ	beta	0.200	0.0050	0.2003	[0.1928, 0.2078]
ρ_{bm}	beta	0.700	0.0100	0.7199	[0.7058, 0.7350]
ρ_{bl}	beta	0.700	0.0100	0.6917	[0.6751, 0.7074]
χ	gamma	1.000	0.0100	0.9970	[0.9794, 1.0117]
σ_e	inv. gamma	0.800	0.5000	1.2401	[0.9173, 1.5575]
σ_a	inv. gamma	0.100	0.0100	0.1015	[0.0809, 0.1211]
σ_{bm}	inv. gamma	0.100	0.0050	0.1068	[0.0939, 0.1232]
σ_{bl}	inv. gamma	0.100	0.0100	0.1188	[0.0988, 0.1407]
σ_ς	inv. gamma	0.010	0.0200	0.0560	[0.0424, 0.0729]

(2) θ_{cb}

A properly estimated θ_{cb} indicates that purchases of middle- and long-term bonds by the central bank affect the one-period nominal interest rate, that is, the policy rate. Whether middle- or long-term bond purchases have a larger impact on the policy rate depends on the estimate of θ_{cb}. If it is larger than 0.5, then the impact of middle-term bond purchases is larger than that of long-term bond purchases. However, in all cases $\tilde{\theta}_{cb}$ is close to 0.5, so that either bond purchase has an almost equal impact.

(3) ρ_ζ

As the parameter ζ_t denotes an adjustment factor of the supply of high-powered money to meet its demand, ρ_ζ implies stickiness of the adjustment factor. In all cases, except for the U.S. policy rule 1 regime, $\tilde{\rho}_\zeta$ suggests that the adjustment factor is highly sticky.

(4) χ

The parameter χ represents the partial impact of changes in the supply of high-powered money on the policy rate. For both countries, $\tilde{\chi}$ is approximately equal to 1. On the other hand, the partial effect of other variables on the policy rate are as follows: $\frac{\partial \hat{r}_t}{\partial \hat{r}_{t-1}} = \rho_{r2}$, $\frac{\partial \hat{r}_t}{\partial \hat{\pi}_t} = (1-\rho_{r2})(\rho_{\pi 2}-\chi)$ and $\frac{\partial \hat{r}_t}{\partial \hat{y}_t} = (1-\rho_{r2})\rho_{y2}$. With the U.S., $\frac{\partial \hat{r}_t}{\partial \hat{r}_{t-1}} = 0.8$, $\frac{\partial \hat{r}_t}{\partial \hat{\pi}_t} = 0.4018$ and $\frac{\partial \hat{r}_t}{\partial \hat{y}_t} = 0.08$; the corresponding values for the U.K. are 0.8, 0.4006 and 0.06, respectively.

Compared to these impacts of other variables, the effect of changes in the supply of high-powered money is significantly high for both countries. This implies that the central banks of both countries could control the policy rate very precisely.

Ⅳ Simulation results

In this section, we illustrate the responses of the one-period rate and the one-period holding returns on middle- and long-term bonds to exogenous monetary and fiscal policy shocks[17]. We define monetary policy shocks as the expansion of the supply of high-powered money and fiscal policy shocks as the augmentation of the outstandings of middle- and long-term bonds.

From the model structure, the former shock is given by increases in ε_ζ by one standard deviation, and the latter shock by rises in ε_{bm} and ε_{bl} by the same standard deviation. To demonstrate fiscal policy shocks, we bring together the responses to rises in ε_{bm} and ε_{bl}, because $\{\varepsilon_{bm,t}\}$ and $\{\varepsilon_{bl,t}\}$ series are assumed to be independent of each other.

Figures 3 and 4 display the responses of the main variables in the U.S. and U.K., respectively. As responses in the U.K. are almost the same as those in the U.S., we explain only responses in the

U.S. below.

1. Policy rule 1 regime

(1) Monetary policy shocks

Many empirical studies show the same standard responses of the economy. A monetary expansion policy generates one-period rate or policy rate reduction, and this reduction induces rises in output, inflation rates and money demand.

It should be noticed that the model has sufficient price stickiness to produce a liquidity effect; that is, one-period rates move in opposite directions to real balances. One-period holding returns on both middle- and long-term bonds decline due to reductions in one-period rates as well as rises in real balances. Declines in both one-period rates and one-period holding returns on bonds are almost the same scale, whereas many empirical studies demonstrate that monetary policy shocks have small or negligible effects on longer-term interest rates. The difference seems to be due to definitions of yields used in studies. Most empirical studies use the constant maturity interest rate, and in that case, the expectations theory is expressed by

$$r_t^{(n)}=\frac{1}{n}E_t[\sum_{i=0}^{n-1} r_{t+i}]+term\ premium,$$

where $r_t^{(n)}$ and r_t denote long-term interest rates with residual maturities of n in period t and one-period interest rates in period t, respectively. According to the above equation, responses of $r_t^{(n)}$ to an impact on r_t would decrease for longer n, even if the impact on r_t is expected to be transmitted to future r_{t+i} on a diminishing scale. On the other hand, as our model uses one-period holding returns on bonds, the expectations theory is described, by our definition of the holding return, as

$$E_t h_t^{(n)}=r_t+term\ premium,$$

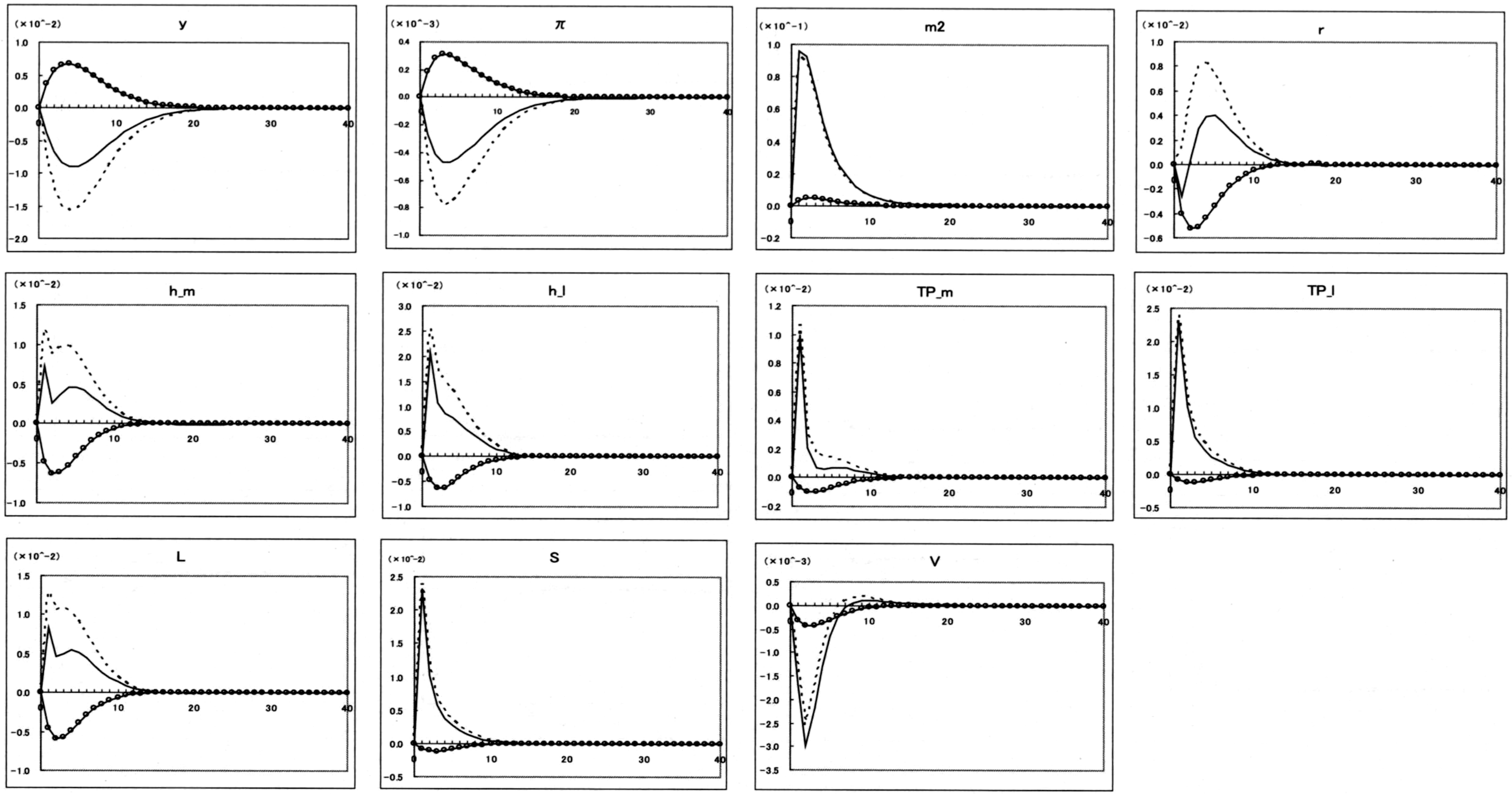
where $h_t^{(n)}$ stands for one-period holding returns on an n-period bond in period t. From the above formula, responses of $h_t^{(n)}$ to an impact on r_t does not change for different periods of n.

However, effects of monetary policy shocks on holding returns are slightly larger for longer-term bonds, inducing lower term premia for such bonds. In this sense, the model reflects the expectation of the monetary authority that a monetary easing policy would decrease longer-term interest rates though reduction in their term premia.

Finally, it is inferred that the level, slope and curvature of yield curves go down; that is, the yield curves turn flatter. This result is inconsistent with that of many empirical studies such as Evans and Marshall [1998, 2007]. One reason for the inconsistency may be the following. As shown in Appendix A, our definition of level, slope and curvature of the yield curve are approximations because we use one-period holding returns, whereas previous studies use accurate

Figure 3. Impulse Responses to the Monetary and Fiscal Shocks (US)

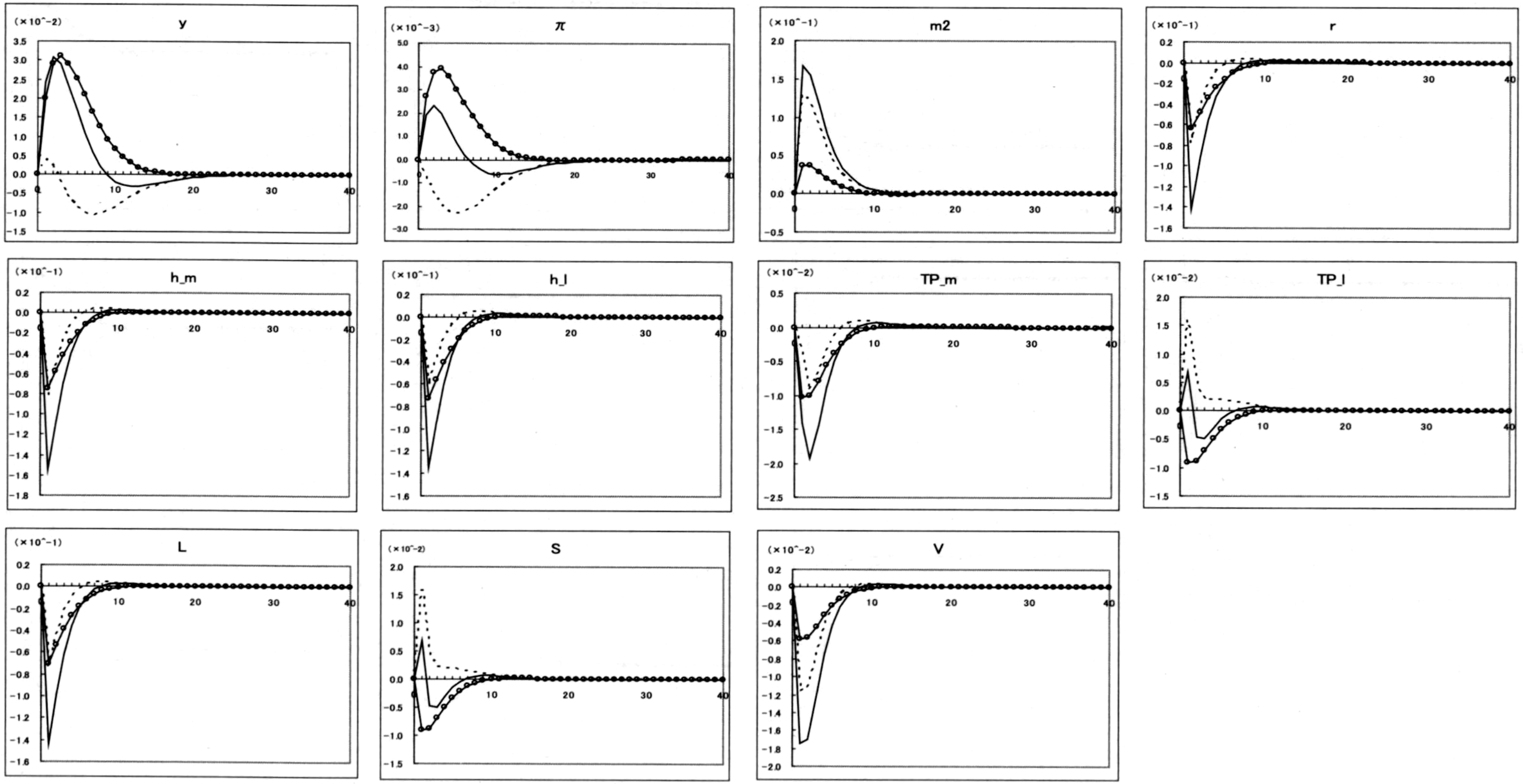
1. Policy Rule 1 Regime



Note: Circle line shows responses to monetary shocks, dotted line to fiscal policy shocks and continuous line to both of the shocks.

2. Policy Rule 2 Regime

Figure 3. Impulse Responses to the Monetary and Fiscal Shocks (US, cont.)



Note: Circle line shows responses to monetary shocks, dotted line to fiscal policy shocks and continuous line to both of the shocks.

definitions based on interest rates. The inconsistency may have been caused by approximation errors.

(2) Fiscal policy shocks

On the one hand, increases in outstandings of middle- and long-term government bonds induce significant growth in demand for money by higher reserve requirements. Although the supply of high-powered money goes up with an increase in bond outstandings, the demand for money grows so sharply as to boost one-period rates rapidly, which leads to a fall in both output and inflation rates.

On the other hand, rises in outstandings of middle- and long-term bonds have a direct influence in pushing up one-period holding returns on these bonds despite growth in the demand for money. In addition, because one-period rates move up, one-period holding returns on both bonds sharply leap up. Consequently, term premia on both holding returns also rise; the yield curve rises and its slope becomes steeper.

(3) Total effects

As explained above, a monetary expansion policy would bring down not only one-period rates but also one-period holding returns on middle- and long-term bonds via declines in each term premium. However, our simulation indicates that, if outstandings of both bonds keep increasing, the lowering effects of monetary policy on one-period holding returns would diminish.

The simulation results suggest that if ε_{ζ}, ε_{bm} and ε_{bl} are all increased by one standard deviation at the same time, one-period rates and one-period holding returns would go up[18]. Furthermore, term premia on both holding returns would rise. As a result, the yield curve rises, and its slope turns steeper.

2. Policy rule 2 regime

(1) Monetary policy shocks

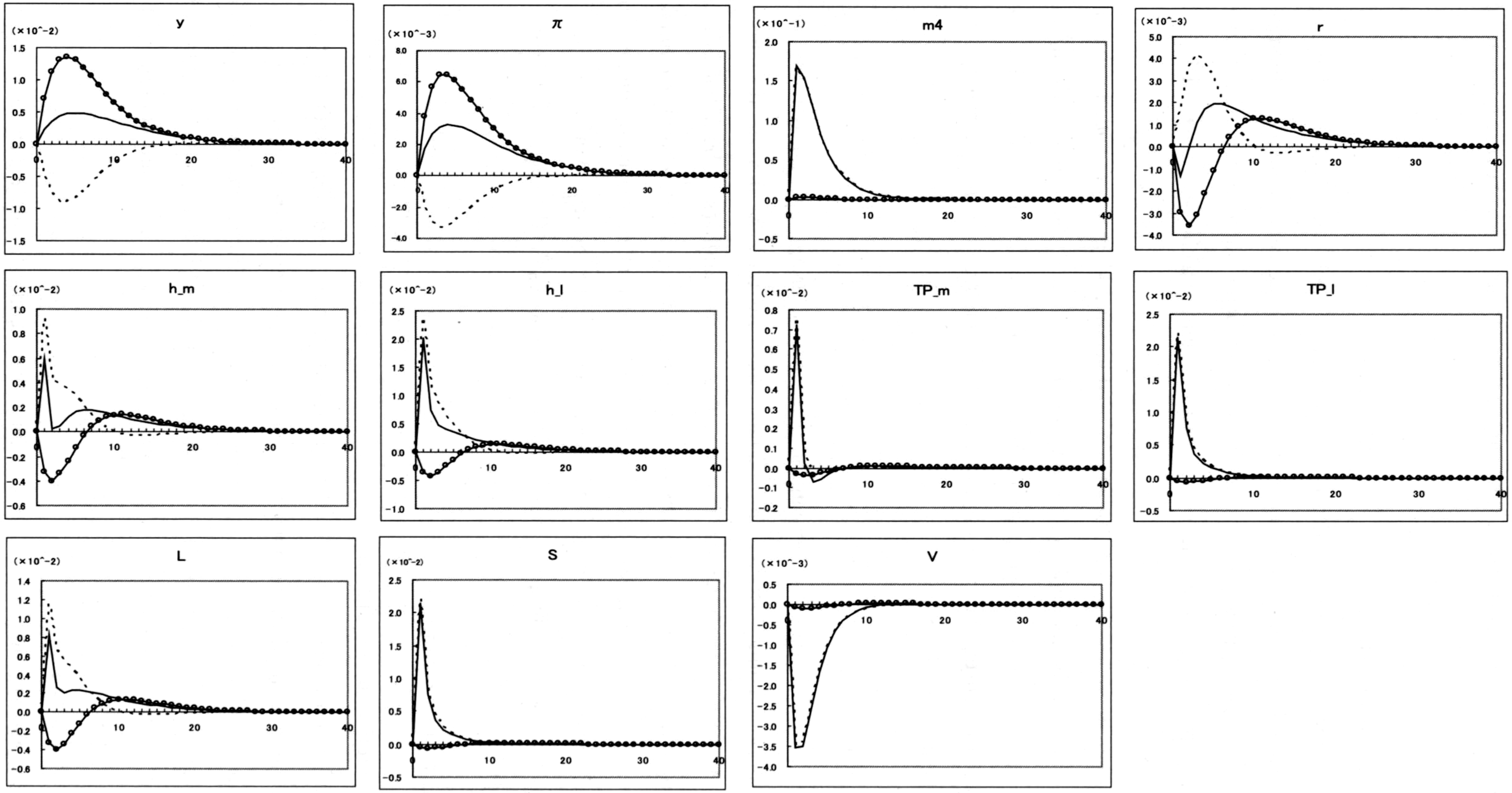
The responses of the economy to monetary expansion policy are very similar to those in the policy rule 1 regime, but the degrees of the responses of all variables are larger in this regime. In brief, monetary policy shocks result in the following influences. First, one-period rates and one-period holding returns on both bonds fall. Second, term premia of holding returns on both bonds decline as intended by the monetary authority. Third, the yield curve drops and its slope is flatter.

(2) Fiscal policy shocks

Increases in outstandings of middle- and long-term bonds would cause the supply of high-pow-

Figure 4. Impulse Responses to the Monetary and Fiscal Shocks (UK)

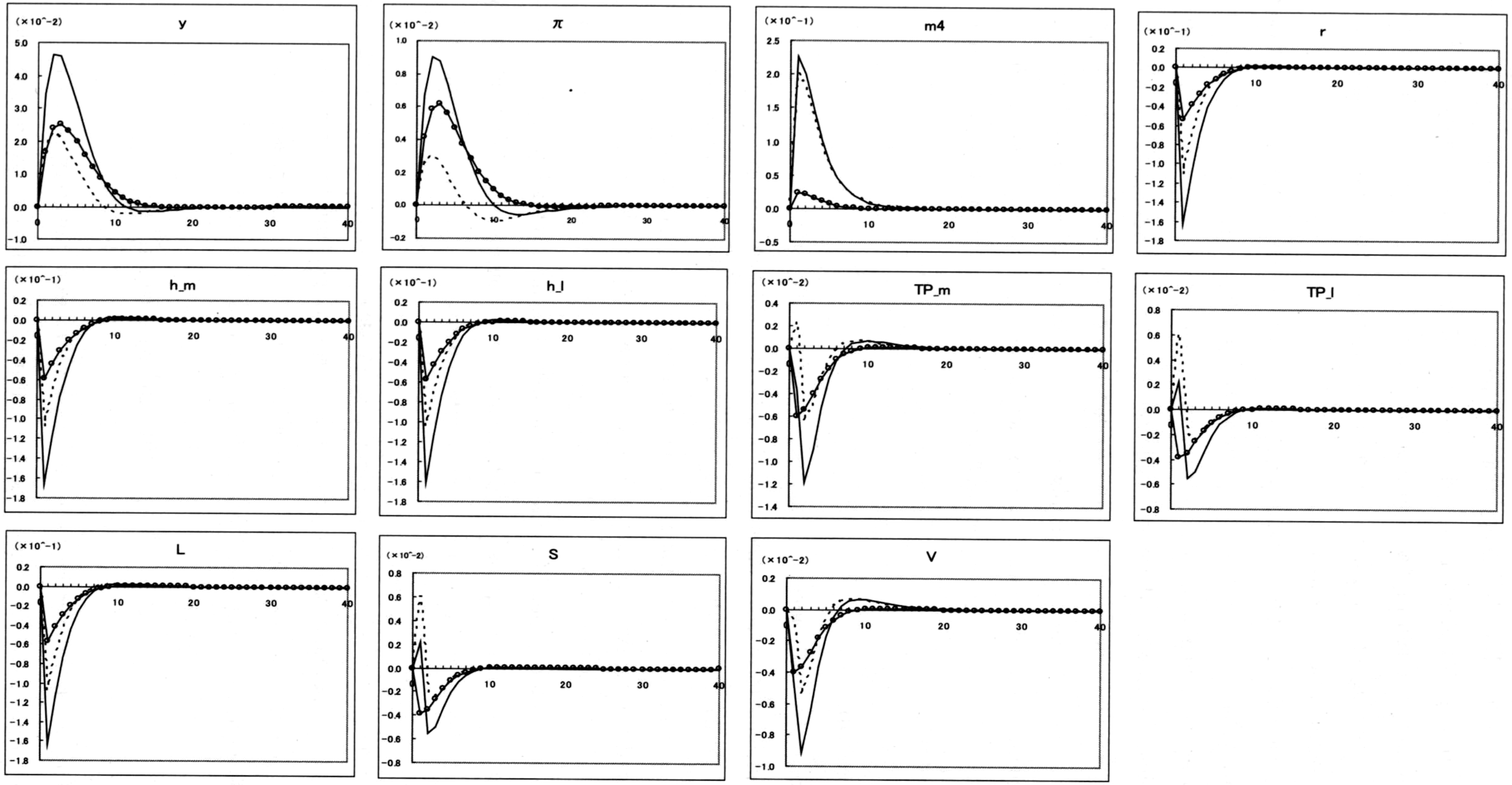
1. Policy Rule 1 Regime



Note: Circle line shows responses to monetary shocks, dotted line to fiscal policy shocks and continuous line to both of the shocks.

Figure 4. Impulse Responses to the Monetary and Fiscal Shocks (UK, cont.)

2. Policy Rule 2 Regime



Note: Circle line shows responses to monetary shocks, dotted line to fiscal policy shocks and continuous line to both of the shocks.

ered money to expand. In the policy rule 2 regime, rises in the supply of high-powered money have much stronger influences in lowering one-period rates or policy rates than in the policy rule 1 regime, because the central bank does not consider the effect of money demand in this regime. Accordingly, although the demand for money sharply increases due to higher reserve requirements, one-period rates sharply decline.

Although increases in the outstandings of both bonds have an impact in raising one-period holding returns on the corresponding bonds, remarkable declines in one-period rates pull down these returns as much a monetary expansion policy would. However the term premia on long-term bonds rise, while those on middle-term bonds decrease.

Increments in government bonds would raise money demand for reserve requirements, and the central bank could expand the supply of high-powered money responding to the increase in money demand. However, without an explicit monetary expansion policy, the declining effects shown above on holding returns as well as on term premia would be too strong to reflect the actual responses.

(3) Total effects

As described above, a monetary expansion policy would lower one-period holding returns on middle- and long-term government bonds via reductions in each term premium. On the other hand, fiscal policy shocks that raise the outstandings of middle- and long-term bonds would also pull down holding returns on bonds because increasing outstandings would expand the supply of high-powered money, even without an explicit monetary expansion policy.

Simulation results suggest that, if all of ε_{ζ}, ε_{bm} and ε_{bl} are all raised by one standard deviation at the same time, both one-period rates and one-period holding returns would rapidly fall. The term premium on long-term bonds rises slightly, whereas that on middle-term bonds decreases sharply. As a result, the yield curve drops remarkably, with a slightly steep shape initially but soon turning flatter.

As mentioned above, simulation results of fiscal policy shocks seem inconsistent with the actual responses of one-period holding returns (i.e., interest rates), and thus discussions on total effects would have little implications as empirical evidence. This suggests that monetary policy framework set as the policy rule 2 regime might be misspecified.

V Concluding remarks

In this paper we have examined the relationship between monetary policy and the yield curve, considering, at the same time, influences of government bond issues on the yield curve.

The basic structure of our model follows the ALSN [2004a] approach, which introduces imperfect substitution between different types of securities, following Tobin [1969]. Operating on the model, we introduce a radical modification to the structure of the monetary policy framework, so the purchases of government bonds by the central bank have some influence on policy rate setting. Following this line, two types of monetary policy rules - policy rule 1 and policy rule 2 - are introduced. However, simulation results on the policy rule 2 regime suggest that the framework of the rule is misspecified.

Simulation on the policy rule 1 regime leads to the following consequence. Monetary expansion policy would lower not only one-period nominal interest rates but also one-period nominal holding returns on middle- and long-term bonds through declines in each term premium. However, if outstandings of both government bonds keep increasing at the same time, the reducing effect of monetary policy on one-period nominal holding returns would be diminished or negated.

The result suggests extremely important implications for the effects of monetary policy. That is, whether the central bank can make an impact on the middle point and/or the long end of the yield curve through purchases of the corresponding government bond would depend on a balance between bonds acquired by the central bank and issues or increases in outstandings of bonds with corresponding terms to maturity.

As stated in section I, the special program in the U.S. appears to have failed to make the intended effects on the yield curve, whereas the U.K. program seems to have attained the purpose of lowering longer-term interest rates. The FRB acquired Treasury securities of 300 billion dollars during the period March 2009 - October 2009 under the program. However, during the same period, Treasury notes, Treasury bonds and TIPS were issued for a total amount of 1,480 billion dollars, and average outstandings of these securities during the period were 4,654 billion dollars. Ratios of securities purchased by the FRB are 20.3% relative to issues, but only 6.4% relative to average outstandings - both rather low. In the U.K., the BEAPFF purchased conventional gilts of 198 billion pounds under the program during the period March 2009 - January 2010. The ratios of purchases to issues and average outstandings of conventional gilts during the period are 108.8% and 31.9%, respectively - much larger than the U.S. ratios. Thus the U.K. program could affect the yield curve. This confirms our empirical results and its implication stated above.

Appendix A: Derivation of equations (23) from equations (21) and (22)

Suppose three bonds with different terms to maturity of n_1, n_2 and n_3 with $n_1 < n_2 < n_3$. When $n_1 = 1$, equation (21) yields

$$h_t^{(1)} = r_t \tag{A1}$$

for all t. In addition, the equation indicates, for $n_k \geq 2$ (k=2, 3),

$$\bar{h}^{(n_k)} = \bar{r}^{(n_k)} \tag{A2}$$

and

$$dr_{t+1}^{(n_k)} = \frac{r_t^{(n_k)} - h_t^{(n_k)}}{n_k - 1}, \tag{A3}$$

where $dr_{t+1}^{(n_k)} \equiv r_{t+1}^{(n_k-1)} - r_t^{(n_k)}$. According to equation (A3), when n_k is very large,

$$dr_{t+1}^{(n_k)} \approx 0. \tag{A4}$$

Meanwhile, using equation (21) and equation (A1), equations (22) can be rearranged by

$$L_t = \frac{1}{3}(h_t^{(n_1)} + h_t^{(n_2)} + h_t^{(n_3)}) + \frac{1}{2}E_t \sum_{k=2}^{3} (n_k - 1)(dr_{t+1}^{(n_k)}),$$
$$S_t = h_t^{(n_3)} - h_t^{(n_1)} + (n_3 - 1)E_t(dr_{t+1}^{(n_3)}), \tag{A5}$$
$$V_t = h_t^{(n_2)} - \frac{1}{2}(r_t + h_t^{(n_3)}) + E_t\left((n_2 - 1)dr_{t+1}^{(n_2)} - \frac{1}{2}(n_3 - 1)dr_{t+1}^{(n_3)}\right).$$

Substituting (A4) into equations (A5) with using (A1) and (A2), we can derive equations (23) approximately.

Appendix B: Model structure

<Households>

$$\hat{\lambda}_t = \frac{(\sigma-1)h}{1-\beta h}\hat{y}_{t-1} + \beta\frac{(\sigma-1)h}{1-\beta h}E_t\hat{y}_{t+1} - \frac{\sigma + (\sigma-1)\beta h^2 - \beta h}{1-\beta h}\hat{y}_t \tag{B1}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t - E_t\hat{\pi}_{t+1} \tag{B2}$$

$$E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{h}_{m,t} - E_t\hat{\pi}_{t+1} = -v_m\left(\frac{\bar{m}}{\bar{b}_m}\right)(\theta\hat{m}_t - \hat{b}_{m,t}) \tag{B3}$$

$$E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{h}_{l,t} - E_t\hat{\pi}_{t+1} = -v_l\left(\frac{\bar{m}}{\bar{b}_l}\right)\{(1-\theta)\hat{m}_t - \hat{b}_{l,t}\} \tag{B4}$$

$$\begin{aligned}\hat{m}_t = {} & \frac{\delta_0}{1+\delta_0(1+\beta)}\hat{m}_{t-1} + \beta\frac{\delta_0}{1+\delta_0(1+\beta)}E_t\hat{m}_{t+1} \\ & -\left(\frac{1-\delta}{\delta}\right)\frac{1}{1+\delta_0(1+\beta)}\hat{e}_{t-1} - \frac{1}{\delta\{1+\delta_0(1+\beta)\}}\hat{\lambda}_t - \frac{\delta_0}{\delta(\bar{R}-1)\{1+\delta_0(1+\beta)\}}\hat{r}_t \\ & -\left(\frac{\bar{R}}{\bar{R}-1}\right)\left[\frac{1}{\delta\{1+\delta_0(1+\beta)\}}\right][\theta v_s(\theta\hat{m}_t - \hat{b}_{m,t}) + (1-\theta)v_l\{(1-\theta)\hat{m}_t - \hat{b}_{l,t}\}],\end{aligned} \tag{B5}$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{e,t} \tag{B6}$$

<Firms>

$$\hat{\pi}_t=\frac{\omega}{1+\omega\beta}\hat{\pi}_{t-1}+\frac{\beta}{1+\omega\beta}E_t\hat{\pi}_{t+1}+\frac{(1-\eta)(1-\beta\eta)}{\eta(1+\omega\beta)}\Theta\hat{mc}_t \quad \text{(B7)}$$

$$\hat{mc}_t=\left\{\frac{\varphi+\alpha}{1-\alpha}+\frac{\sigma+(\sigma-1)\beta h^2-\beta h}{1-\beta h}\right\}\hat{y}_t-\frac{(\sigma-1)h}{1-\beta h}\hat{y}_{t-1} -\beta\frac{(\sigma-1)h}{1-\beta h}E_t\hat{y}_{t+1}-\frac{\varphi+\alpha}{1-\alpha}\hat{a}_t \quad \text{(B8)}$$

$$\hat{a}_t=\rho_a\hat{a}_{t-1}+\varepsilon_{a,t} \quad \text{(B9)}$$

<The central bank>

$$\varsigma_t=\rho_\varsigma\varsigma_{t-1}+\varepsilon_{\varsigma,t} \quad \text{(B10)}$$

(Policy rule 1 regime)

$$\hat{r}_t=\rho_{r1}\hat{r}_{t-1}+(1-\rho_{r1})[\rho_{\pi1}\hat{\pi}_t+\rho_{y1}\hat{y}_t+\rho_\mu\{\widehat{m}_t-\theta_{cb}\hat{b}_{m,t}-(1-\theta_{cb})\hat{b}_{l,t}-\varsigma_t\}] \quad \text{(B11)}$$

(Policy rule 2 regime)

$$\hat{r}_t=\rho_{r2}\hat{r}_{t-1}+(1-\rho_{r2})(\rho_{\pi2}\hat{\pi}_t+\rho_{y2}\hat{y}_t) -\chi\{\theta_{cb}(\hat{b}_{m,t}-\rho_\varsigma\hat{b}_{m,t-1})+(1-\theta_{cb})(\hat{b}_{l,t}-\rho_\varsigma\hat{b}_{l,t-1})+\hat{\pi}_t+\varepsilon_{\varsigma,t}\} \quad \text{(B12)}$$

<The government>

$$\hat{b}_{m,t}=\rho_{bm}\hat{b}_{m,t-1}+\varepsilon_{bm,t} \quad \text{(B13)}$$

$$\hat{b}_{l,t}=\rho_{bl}\hat{b}_{l,t-1}+\varepsilon_{bl,t} \quad \text{(B14)}$$

<Yield curve>

$$\hat{L}_t=\frac{1}{3}(\hat{r}_t+\hat{h}_{m,t}+\hat{h}_{l,t}) \quad \text{(B15)}$$

$$\hat{S}_t=\hat{h}_{l,t}-\hat{r}_t \quad \text{(B16)}$$

$$\hat{V}_t=\hat{h}_{m,t}-\frac{1}{2}(\hat{r}_t+\hat{h}_{l,t}) \quad \text{(B17)}$$

$$\hat{TP}_{m,t}=\hat{h}_{m,t}-\hat{r}_t \quad \text{(B18)}$$

$$\hat{TP}_{l,t}=\hat{h}_{l,t}-\hat{r}_t \quad \text{(B19)}$$

Appendix C: Data and data sources

1. Output: Y_t

(1) U.S.: Disposable personal income (seasonally adjusted), *U.S. Department of Commerce: Bureau of Economic Analysis, Personal Income and Outlays.*

(2) U.K.: Estimated series comprising of the indexes of production (IOP) and services (IOS) according to these GDP weight in 2005, where the properties of IOP and IOS are as follows.

(a) IOP: 2005=100, seasonally adjusted, GDP weight=172.2/1000;

(b) IOS: 2005=100, seasonally adjusted, GDP weight=758.5/1000.

The data source is *Office for National Statistics* (ONS).

2. Price: P_t

(1) U.S.: Consumer price index: all items (1982-84=100, seasonally adjusted), *U.S. department of Labor: Bureau of Labor Statistics, Consumer Price Index.*

(2) U.K.: Consumer price index: all items (2005=100, not seasonally adjusted), *ONS.* Seasonally adjusted series of the inflation rate is estimated by $\pi_t = (\ln P_t - \ln P_{t-12})/12$.

3. Money supply (money demand): M_t

(1) U.S.: M2 money stock (seasonally adjusted), *Board of Governors of the Federal Reserve System* (FRB), *H.6 Money Stock Measures.*

(2) U.K.: M4 money stock (seasonally adjusted), *ONS.*

4. One-period interest rate: r_t

(1) U.S.: Monthly average of federal funds rate, *FRB, H.15 Selected Interest Rates.*

(2) U.K.: Monthly average of official bank rate, *Bank of England* (BOE), *Interactive Database.*

5. Middle-term interest rate: $r_{m,t}$

(1) U.S.: 5-year zero-coupon yield (nominal, end of month), *The US Real Term Structure of Interest Rates with Implicit Inflation Premium* presented by J. Huston McCulloch (http://econ.ohio-state.edu/jhm/ts/ts.html).

(2) U.K.: 5-year zero-coupon yield (nominal, end of month), *BOE, Financial Market Data, Estimates of UK Yield Curves.*

6. Long-term interest rate: $r_{l,t}$

(1) U.S.: 10-year zero-coupon yield (nominal, end of month), *The US Real Term Structure of Interest Rates with Implicit Inflation Premium* presented by J. Huston McCulloch (http://econ.ohio-state.edu/jhm/ts/ts.html).

(2) U.K.: 10-year zero-coupon yield (nominal, end of month), *BOE, Financial Market Data, Estimates of UK Yield Curves.*

7. Middle-term government bond outstandings: $B_{m,t}$

(1) U.S.: Treasury securities outstanding (marketable, terms to maturity of 1-7 years, not seasonally adjusted, end of month), *U.S. Department of the Treasury: Bureau of the Public Debt, Monthly Statement of the Public Debt.*

(2) U.K.: Gilts outstanding (terms to maturity of 1-7 years, not seasonally adjusted, end of month), *Heriot-Watt/Faculty and Institute of Actuaries Gilt Database* (http://www.ma.hw.ac.uk/~andrewc/gilts/).

8. Long-term government bond outstandings: $B_{l,t}$

(1) U.S.: Treasury securities outstanding (marketable, terms to maturity of over 7 years, not seasonally adjusted, end of month), *U.S. Department of the Treasury: Bureau of the Public Debt, Monthly Statement of the Public Debt.*

(2) U.K.: Gilts outstanding (terms to maturity of over 7 years, not seasonally adjusted, end of month), *Heriot-Watt/Faculty and Institute of Actuaries Gilt Database* (http://www.ma.hw.ac.uk/~andrewc/gilts/).

Notes

1) The Federal Reserve Bank of New York [2010] (p. 7) states "Indeed, over the course of the program the Desk (Trading Desk of the Federal Reserve Bank of New York) purchased $242 billion in nominal Treasury securities maturing in 2 to 10 years, $42 billion maturing in 10 to 30 years, $11 billion maturing in 1 to 2 years and $5 billion in TIPS." On average, after 2000 to 2009, issue amounts of Treasury bonds and TIPS with maturities of more than 10 years account for only 4.6% of whole issue amount of Treasury notes, Treasury bonds and TIPS, which may be one of reasons the program mainly targeted on Treasury securities with residual maturities of 2 to 10 years.

2) The BEAPFF is a subsidiary company of the Bank of England (BOE).

3) As for the latter purpose, see Bank of England [2009] and Benford et al. [2009] for the U.K. For the U.S., see Gagnon et al. [2010], the December 2008 minutes of the Federal Open Market Committee, and speeches of Ben S. Brenanke (chairman of the FRB) on December 1, 2008 (http:// www. federalreserve.gov/newsevents/speech/bernanke20081201a.htm) and of Donald L. Kohn (vice chairman of the FRB) on April 18, 2009 (http://www.federalreserve.gov/newsevents/speech/kohn20090418a.htm).

4) As there are a number of studies, the literature listed in each group is restricted to recent works on the U.S. and U.K. economy.

5) Kuttner [2006] presents a brief review of previous studies in the group.

6) One-period bond holdings of $B_{s,t}$ are not included in the cost function AC_t. From Tobin's [1969] viewpoint, to induce the public to hold an increase in the relative supply of the more illiquid assets, the spread between illiquid and liquid assets should be bid up. Accordingly, it would probably be closer to Tobin's position to include not only money but also one-period bonds as liquid assets in the function, in order to define liquid assets more broadly. However high-powered money certainly belongs in the total amount of money, and this narrow definition of liquidity is sufficient to capture an essential feature of Tobin's framework, that is, the extra channel of monetary policy recently invoked by certain central banks, e.g. FRB, BOE, the European Central Bank and the Bank of Japan.

7) Although the model does not explicitly treat financial institutions such as banks, it is assumed that households purchase financial assets through the institutions. In this context, it is reasonable to suppose that the institution would provide against unexpected money demand.

8) Combining equations (8) and (11) yields the right-hand side of (11)$=\left(\frac{\lambda_t}{P_t}\right)\left(1-\frac{1}{R_t}\right)$. This equation indicates a relation linking nominal interest rates to the marginal rate of substitution between money and wealth, and thus implies that expectations of real income and nominal rates matter for contemporaneous portfolio decisions.

9) For derivation of equations (23) from equations (21) and (22), see Appendix A.

10) In equation (24), $Y_t(j)$ and $N_t(j)$ are interpreted as ones per capital in period t for firm j.

11) If we explicitly consider that the central bank purchases government bonds, we could derive equation (51) as follows. Define the amount of bonds acquired by the central bank as $B_{cb,k,t}$ (k=m, l), and the supply function as $HM_t^S=\kappa_{cb,S}B_{cb,m,t}^{\theta_{cb}}B_{cb,l,t}^{1-\theta_{cb}}$, where $\kappa_{cb,S}\equiv\frac{\overline{HM^S}}{\overline{B}_{cb,m,t}^{\theta_{cb}}\overline{B}_{cb,l,t}^{1-\theta_{cb}}}$. Furthermore, assume the relationship between $B_{cb,k,t}$ and $B_{k,t}$ as follows: $B_{cb,k,t}=\exp(\kappa_{k,t})B_{k,t}$, where $\kappa_{k,t}=\rho_{\kappa,k}\kappa_{k,t-1}+\varepsilon_{\kappa,k,t}$ with $\rho_{\kappa,k}\in(0, 1)$ and $\varepsilon_{\kappa,k,t}\sim i.i.d.(0, \sigma_{\kappa,k}^2)$. Substituting these equations into

the above supply function and rearranging the resulting equation, $HM_t^S = \kappa_{cb,S} \exp(\theta_{cb}\kappa_{m,t} + (1-\theta_{cb})\kappa_{l,t}) B_{cb,m,t}^{\theta_{cb}} B_{cb,l,t}^{1-\theta_{cb}}$. Defining $\kappa_{cb,S}\exp(\theta_{cb}\kappa_{m,t} + (1-\theta_{cb})\kappa_{l,t})$ in this equation as $\kappa_{cb}\exp(\varsigma_t)$ yields equation (51).

12) For the U.S., ALSN [2004a] infer φ as around 1/1.6 from the estimate of $(\varphi+\alpha)/(1-\alpha)$ (parameter χ in their context) =1.36. Based on the estimate, α is inferred as 0.3114. These inferred values of α and φ for the U.S. conform to our setting of α=0.36 and φ=1, as shown below.

13) In conducting Bayesian MCMC estimation, we make use of the DYNARE software (version 4.04) for MATLAB.

14) In addition, results from posterior maximization indicate that the estimated standard deviation of parameters is low enough to be statistically significant.

15) In the U.S., the average annual growth rate of CPI was 2.56% for the period January 2000 – October 2009, while it was 2.94% in the 1990s and 4.98% in the 1980s. In the U.K., it was 1.89% for the same period in the 2000s, whereas it was 3.14% in the 1990s.

16) DiCecio and Nelson [2007] estimate the value of η as 0.9371 with β set at $(1.04)^{-1/4}$. Using their result with our estimates of $\tilde{\omega}$ = 0.6 and $\tilde{\Theta}$ = 0.2, the slope of NKPC is calculated as 6.19×10^{-4} for the U.K. This slope estimate is close to our estimates.

17) As described in section II, the model defines all variables, except the interest rate and the holding returns, as real ones, so that we will omit the words "real" and "nominal," without confusion, in this section.

18) We assume that $\{\varepsilon_{\varsigma,t}\}$ series are independent of both $\{\varepsilon_{bl,t}\}$ and $\{\varepsilon_{bm,t}\}$ series.

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